Geometry dependent suppression of collective quantum jumps in Rydberg atoms

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Abstract

We consider N driven, damped Rydberg atoms in different spatial arrangements. Treating the atoms as two-level systems we model the coupling to the environment via the Lehmberg-Agarwal master equation which interpolates between fully independent and fully collective spontaneous emission depending on the specific locations of the atoms. We also include a collective dipole-dipole energy shift in the excited Rydberg state which leads to collective quantum jumps in the atomic excitation when the system is driven off resonance. We show that the quantum jumps are suppressed as the system makes a transition from independent to collective emission as the spacing of a linear array of atoms is decreased below the emission wavelength.

1. System

- The large dipole moment of Rydberg atoms can lead to a Rydberg blockade [1] and collective quantum jumps [2] with applications for quantum information processing [3].
- We consider a system of N driven, damped two-level atoms. Making the electric dipole and rotating wave approximations the Hamiltonian in the interaction picture is ($\hbar = 1$) [2]

$$H = \sum_{j=1}^{N}\left[-\Delta|e\rangle\langle e|_j + \frac{\Omega}{2}(\sigma_{j+} + \sigma_{j-})\right] + \frac{V}{N-1}\sum_{j<k=1}^{N}|e\rangle\langle e|_j \otimes |e\rangle\langle e|_k \tag{1}$$

where σ_{j-} is the Pauli lowering operator for the jth atom, Δ is the detuning of the driving laser from the atomic resonance, Ω is the Rabi frequency, and V is a constant that accounts for the long range interaction between the Rydberg atoms in their excited states.

- Considering a system of two atoms the double excited state is shifted upward in energy by the V term in the Hamiltonian as shown in Fig. 1. Shifts between multi atom systems vary in size as a result of their geometric configurations.

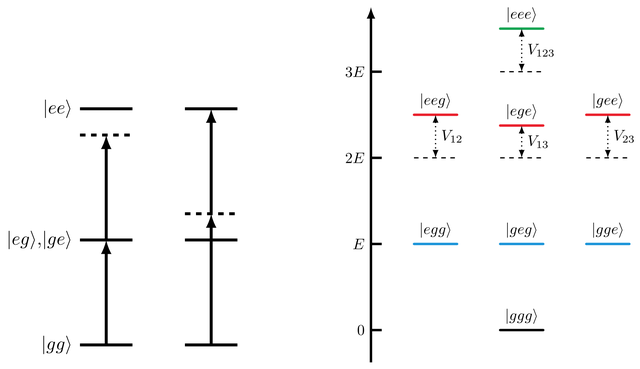

Figure 1: *When $\Delta = 0$ the double excited state is shifted out of resonance and excitation is suppressed as seen on the right. This is the familiar Rydberg blockade effect. When $\Delta = V/2$ the double excited state is on resonance via a two photon transition. The shifts are also dependent on the arrangement of atoms as seen on the right.*

- We include damping using a master equation derived in the Born-Markov approximation:

$$\dot{\rho} = -i\,[H,\rho] + \frac{\gamma}{2}\sum_{j=1}^{N}(2\sigma_{j-}\rho\sigma_{j+} - \rho\sigma_{j+}\sigma_{j-} - \sigma_{j+}\sigma_{j-}\rho) \tag{2}$$

describes independent spontaneous emission and

$$\dot{\rho} = -i\,[H,\rho] + \frac{\gamma}{2}(2J_-\rho J_+ - \rho J_+ J_- - J_+ J_-\rho) \tag{3}$$

describes collective spontaneous emission where $J_- = \sum_{j=1}^{N}\sigma_{j-}$.

- We compare the average jump length of collective quantum jumps for various separation distances. Limiting cases were also checked for fully independent and fully collective collapse operators.

2. Methods

- We numerically solve the master equation for the system by means of a quantum trajectory simulation implemented using the Quantum Toolbox in Python (QuTiP) [4, 5].
- The intermediate case was accomplished by introducing a γ_{ij} term that was used to define the spontaneous emission of a set of dipoles.[6, 7]

$$\gamma_{ij} = \gamma\frac{3}{2}\left\{[1-(\hat{d}\cdot\hat{r}_{ij})^2]\frac{\sin\xi_{ij}}{\xi_{ij}} + [1-3(\hat{d}\cdot\hat{r}_{ij})^2]\left(\frac{\cos\xi_{ij}}{\xi_{ij}^2} - \frac{\sin\xi_{ij}}{\xi_{ij}^3}\right)\right\}$$

where $\xi_{ij} \equiv k_0 r_{ij} = 2\pi r_{ij}/\lambda_0, \qquad \mathbf{r}_{ij} = r_{ij}\hat{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$

- The eigenvalues, λ_i, and eigenvectors, $\vec{v}_i$, where calculated for the γ_{ij} matrix and used to determine the collapse operators for our system by $J_i = \sqrt{\lambda_i}\sum_j^N \vec{v}_{ij}\sigma_{j-}$ where J_i is the ith collapse operator.
- For a system that is separated by a large distance, the eigenvalues of our γ_{ij} matrix approach one leaving our system like independent emission. At the other limit, where our distances become very small the eigenvalues of the γ_{ij} matrix would approach approach zero expect for one which approaches N.

Independent case: $\begin{pmatrix}1 & 0 & \dots\\ 0 & 1 & \dots\\ \vdots & \vdots & \ddots\end{pmatrix} \Longrightarrow$ all $\lambda_i \to 1$ Collective case: $\begin{pmatrix}1 & 1 & \dots\\ 1 & 1 & \dots\\ \vdots & \vdots & \ddots\end{pmatrix} \Longrightarrow$ one $\lambda \to N$

3. Quantum Trajectory Results

- To explore the state of a single quantum trajectory a Monte Carlo method was implemented in QuTip. The average energy of the system was computed as a function of time. This simulation was run for various separation lengths. As the atoms get closer together their emission type becomes more like collective spontaneous emission and the quantum jumps seem to disappear.

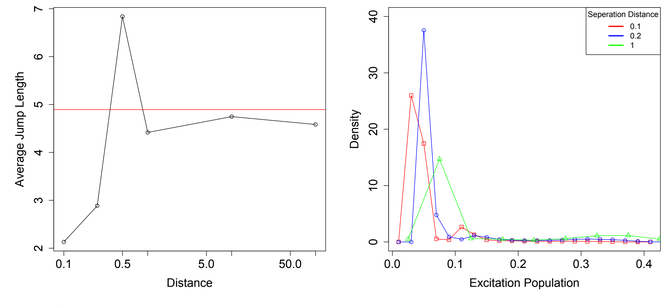

Figure 2: *On the left we see the average jump length approaches the independent jump length for large values of R. On the right we plot the average excitation population to show the bistability. The parameters for all of the runs are: $N = 16, \Delta = 3.5, \Omega = 1.5, V = 10$*

- Various trajectories were run and the average jump length calculated as a characteristic value to analyze bistability.
- The energy distribution was also useful in analyzing the bistability for our system

4. Quantum Jumps

- The excitation population is show as the emission type changes. As the atoms move closer together they emit collectively, and they jumps are inhibited

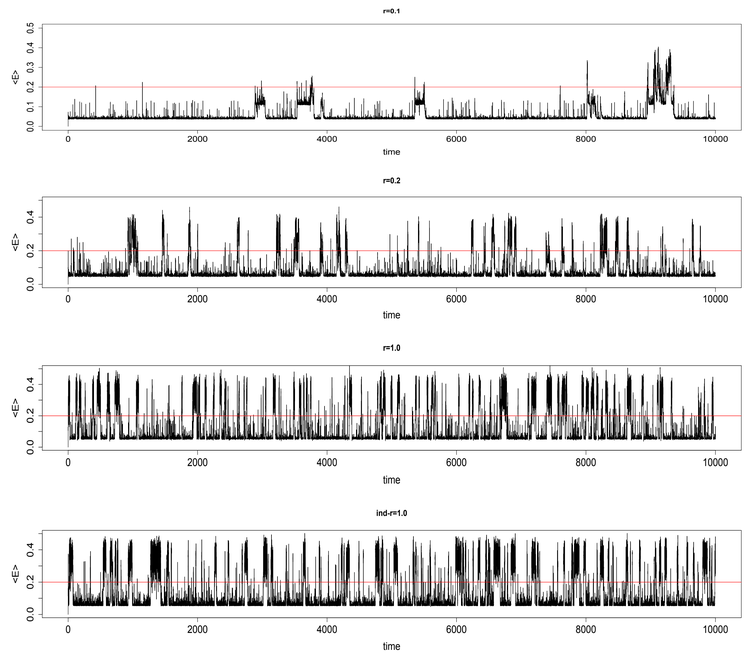

Figure 3: *We plot the excitation population for 16 atoms for various separation distances. Notice how for small R the jumps are greatly diminished. Parameters used $N = 16, \Delta = 3.5, \Omega = 1.5, V = 10$*

5. Future Work

- We plan to explore the properties of these collective quantum jumps as a function of geometry as well as distance. As the atoms move closer together their emission becomes more collective which inhibits the collective jumps. The larger the shift in the energies the more likely there is for jumps to occur. The V_{ij} increases as distance is shorted. We want to see for what conditions are jumps still show as distance is varied and V_{ij} competes with the collective emission

References

[1] E. Urban, T. A. Johnson, T. Henage, L. Isenhower, D. D. Yavuz, T. G. Walker, and M. Saffman, "Observation of rydberg blockade between two atoms," *Nature Physics*, vol. 5, pp. 110–114, 2009.

[2] T. E. Lee, H. Haffner, and M. C. Cross, "Collective quantum jumps of rydberg atoms," *Phys. Rev. Lett.*, vol. 108, p. 023602, 2012.

[3] T. G. Walker and M. Saffman, "Entanglement of two atoms using rydberg blockade," *Adv. At. Mol. Opt. Phys.*, vol. 61, p. 81, 2012.

[4] J. R. Johansson, P. D. Nation, and F. Nori, "Qutip: An open-source python framework for the dynamics of open quantum systems," *Comp. Phys. Comm.*, vol. 183, pp. 1760–1772, 2012.

[5] J. R. Johansson, P. D. Nation, and F. Nori, "Qutip 2: A python framework for the dynamics of open quantum systems," *Comp. Phys. Comm.*, vol. 184, p. 1234, 2013.

[6] R. H. Lehmberg, "Radiation from an N-atom system. I. General formalism," *Phys. Rev. A*, vol. 2, pp. 883–888, September 1970.

[7] G. S. Agarwal, "Master-equation approach to spontaneous emission," *Phys. Rev. A*, vol. 2, pp. 2038–2046, 1970.