ABSTRACT

SUPPRESSION OF COLLECTIVE QUANTUM JUMPS
OF RYDBERG ATOMS DUE TO COLLECTIVE SPONTANEOUS EMISSION
FROM ATOMS IN FREE SPACE

by Eitan Jacob Lees

We consider N driven, damped Rydberg atoms in different spatial arrangements. Treating the atoms as two-level systems we model the coupling to the environment via the Lehmberg-Agarwal master equation which interpolates between fully independent and fully collective spontaneous emission depending on the specific locations of the atoms. We also include a collective dipole-dipole energy shift in the excited Rydberg state which leads to collective quantum jumps in the atomic excitation when the system is driven off resonance. We show that the quantum jumps are suppressed as the system makes a transition from independent to collective emission as the spacing of a linear array of atoms is decreased below the emission wavelength.

SURPRESSION OF COLLECTIVE QUANTUM JUMPS
OF RYDBERG ATOMS DUE TO COLLECTIVE SPONTANIOUS EMISSION
FROM ATOMS IN FREE SPACE

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Contents

List of Figures

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Chapter 1

Introduction

One of the interesting phenomena of quantum mechanics is the concept of a quantum jump, where a system will switch between two discrete states. This behavior is most commonly seen with respect to the electron transition between energy levels of an atom. I have been studying the collective quantum jumps of Rydberg atoms. The large dipole moment of the highly excited Rydberg atom causes a shift in the energy levels of surrounding atoms. This shift suppresses the excitation of neighboring atoms when driven on resonance and this is known as the Rydberg blockade [1] . My area of research pertains to collections of Rydberg atoms that exhibit quantum jumps when driven off resonance. The collective behavior of these many body quantum jumps is unique and an area of interest.

1.1   Quantum Jumps

At the beginning of the 20th century there were many new ideas brewing on the nature of atoms and the physics of the very small. In the development of a basic model of an atom Niels Bohr, with the help of Ernest Rutherford, hypothesised that electrons orbit the nucleus in discrete circular paths [2]. A major hypothesis of his theory was that the electrons make jumps between orbits while absorbing or emitting the appropriate amount of energy. Though this description of an atom is now obsolete due to its simplicity, the Bohr model was birthplace for the idea of the quantum jump.

The detailed description of the interaction of light and matter was one of the major succeses during the development of quantum mechanics. One of the first theories of this process was a statistical mechanics approach by Albert Einstein in which he defined his famous A and B coefficients [3]. Einstein proposed that there were three factors pertaining to the interaction of light and matter to produce spectral lines. The A coefficient represented the spontaneous emission of an atom, the B coefficient represented the absorption of a

photon as well as a new concept which he called stimulated emission. Stimulated emission was regarded in the context of thermodynamics as a negative absorption. This is sometimes regarded as the birthplace of the laser, and is considered the earliest mention of stimulated emission [4]. Einstein utilized the theory of blackbody radiation to define a ratio of these new coefficients. He set the ratio of A/B equal to the equilibrium radiation density per unit volume per unit frequency, $\varrho(\nu) = 8\pi h \nu^3/c^3$, connecting his new theory with the rich theories of thermodynamics.

The predictions of quantum mechanics have often been regarded as bizarre or unconventional but through careful experiments the theory has passed all of the tests. The quantum jump remained in the context of statistical mechanics for many years until a single jump was directly observed by trapping a single ion [5, 6]. An ion was cooled and localized using two collinear laser beams to a region of $< 1\mu$m and then a shelving process was used to observe the jump. The $6^2S_{1/2}$-$6^2P_{1/2}$ and the $5^2D_{3/2}$-$6^2P_{1/2}$ were driven by two dye lasers. In order to observe the jump the atom needed to be in the $5^2D_{5/2}$ state. This was achieved by a filtered lamp that excited the atom to the $6^2P_{3/2}$ state and then it could decay into the $5^2D_{5/2}$ state. The $6^2P_{1/2}$-$6^2S_{1/2}$ transition was monitored using a photomultiplier tube of known quantum efficiency. Clear jumps can be observed between two quantum states. They were shown to happen in a single jump rather than occurring in steps which implied that a single ion was trapped. The direct observation of quantum jumps further solidified the foundations of quantum theory and also pushed the experimental techniques to new heights. The dynamics of a single atom as it absorbs and emits light is a subject that is at the heart of quantum optics. The formalism outlined to describe an open quantum system as it evolves is known as a quantum trajectory [7]. Quantum Optics currently has both the theoretical framework and the experimental techniques to explore the most fundamental questions about how light and matter interact.

My work is focused specifically on the dynamics of collections of Rydberg atoms. In recent years Rydberg atoms have been promising candidates for qubits in quantum computers [8, 9] and offer exciting opportunities for future technologies. A theoretical paper by Tony E. Lee, H. Häffner, and M.C. Cross [10] studied the dynamics of collective Rydberg quantum jumps though they have not been experimentally observed yet. They showed that there is a dispersive bistability for collections of Rydberg atoms and that quantum fluctuations drive the jumps between these stable states. The two level optical Bloch equations were solved under the assumption of a mean field theory to better understand the source of the bistability. Quantum trajectories were utilized to explore the characteristics of these jumps including jump length, excitation population density and excitation population vs emission number. They showed that the number of successive photon emissions was an indicator that

a collective jump was going to occur. If successive photons were emitting then the system would jump to a "Bright" state and with the absence of emissions the system would return to a "Dark" state. I have built on this study by introducing a geometric components into the Hamiltonian and running my simulation for various geometric arrangements of atoms. Whether the system emits photons through fully independent spontaneous emission or fully collective spontaneous emission has shown to have a major effect on the dynamics of the system which has been explored.

1.2 Thesis Outline

The main focus of this thesis is to explore the properties of collections of Rydberg atoms but first the theoretical background of open quantum systems is explained. The Lindblad form of the master equation is derived and also a brief description of the algorithms of quantum trajectories is given. Next the specifics of my model are detailed including the implementation of an intermediate model for the emission type. An over view of the QuTiP framework is explained in the context of my project. Results are presented from the probe spectra, cross correlations, and quantum trajectories explored for my research. Statistics were performed on the quantum trajectories to characterize the collective quantum jumps such as average jump length and total jump count. Finally a conclusion is provided suggesting future plans for the project. An appendix is also included detailing the code used to produce figures and make various simulations.

Chapter 2

Dissipative Quantum Systems

Many systems are unavoidably tied to an environment which leads to dissipation. The methods to incorporate dissipation into quantized systems were an area of much interest in the development of quantum optics. The methods and problems approached in the study of open quantum systems is related to the study of spontaneous emission. We are going to use these tools to model independent and collective spontaneous emission.

2.1 Interaction Hamiltonian

The Hamiltonian for a system coupled to the environment is written as a sum of three terms,

$$H = H_s + H_R + H_{sR} \tag{2.1}$$

where H_s represents the system of interest, H_R represents the environment, and H_{sR} represents the coupling between the two. Let the term $\chi(t)$ represent the density operator for the combination of our system and environment. Thus our density matrix for the system alone is expressed as a trace over our environmental variables $\rho(t) \equiv \text{Tr}_R[\chi(t)]$. This allows us to calculate the expectation value of a system operator $\hat{O}$ such that

$$\langle \hat{O} \rangle = \text{Tr}_{sR}[\hat{O}\chi(t)] = \text{Tr}_s\{\hat{O}\,\text{Tr}_R[\chi(t)]\} = \text{Tr}_s[\hat{O}\rho(t)] \tag{2.2}$$

requiring only the information of $\rho(t)$ instead of the full $\chi(t)$. Now our goal is to solve the Schrödinger equation,

$$\dot{\chi} = -\frac{i}{\hbar}[H, \chi] \tag{2.3}$$

with this coupled system, eq. (2.1), and obtain an expression for $\rho(t)$. Before we directly solve eq. (2.3) we need to consider a different perspective.

In quantum mechanics there are several ways to deal with the time dependence of the system. The Schrödinger picture is when all of the time dependence is wrapped up in the state vector and the operators are constant in time. The Heisenberg picture is when the state vectors are constant in time and the operators take on all of the time dependence. We will need to share the time dependence between the operators and the state vector. This is known as the interaction picture. The motivation for this different perspective is that $H_s + H_R$ generate time evolution on a short time scale whereas H_{sR} moves relatively slowly so it is useful to separate these time scales. Going to the interaction picture the state becomes

$$\tilde{\chi}(t) = e^{\frac{i}{\hbar}(H_s+H_R)t}\chi(t)e^{-\frac{i}{\hbar}(H_s+H_R)t} \tag{2.4}$$

which can be used to solve for dynamics. The Schrödinger equation in the interaction picture now becomes

$$
\begin{aligned}
\dot{\tilde{\chi}}(t) &= e^{\frac{i}{\hbar}(H_s+H_R)t}\frac{d}{dt}\left[\chi e^{-\frac{i}{\hbar}(H_s+H_R)t}\right] \\
&\quad + \frac{i}{\hbar}(H_s+H_R)e^{\frac{i}{\hbar}(H_s+H_R)t}\chi e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&= e^{\frac{i}{\hbar}(H_s+H_R)t}\left[-\frac{i}{\hbar}\chi(H_s+H_R)e^{-\frac{i}{\hbar}(H_s+H_R)t} + \dot{\chi}e^{-\frac{i}{\hbar}(H_s+H_R)t}\right] \\
&\quad + \frac{i}{\hbar}(H_s+H_R)e^{\frac{i}{\hbar}(H_s+H_R)t}\chi e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&= -\frac{i}{\hbar}\tilde{\chi}(H_s+H_R) + e^{\frac{i}{\hbar}(H_s+H_R)t}\dot{\chi}e^{-\frac{i}{\hbar}(H_s+H_R)t} + \frac{i}{\hbar}(H_s+H_R)\tilde{\chi} \\
&= \frac{i}{\hbar}\left[H_s+H_R,\tilde{\chi}\right] + e^{\frac{i}{\hbar}(H_s+H_R)t}\left(\frac{-i}{\hbar}\right)[H,\chi]e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&= \frac{i}{\hbar}\left[H_s+H_R,\tilde{\chi}\right] + e^{\frac{i}{\hbar}(H_s+H_R)t}\left(\frac{-i}{\hbar}\right)[H_s+H_R+H_{sR},\chi]e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&= -\frac{i}{\hbar}e^{\frac{i}{\hbar}(H_s+H_R)t}\left(H_{sR}\chi - \chi H_{sR}\right)e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&= -\frac{i}{\hbar}e^{\frac{i}{\hbar}(H_s+H_R)t}H_{sR}\left(e^{\frac{i}{\hbar}(H_s+H_R)t}e^{\frac{-i}{\hbar}(H_s+H_R)t}\right)\chi e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&\quad + \frac{i}{\hbar}e^{\frac{i}{\hbar}(H_s+H_R)t}\chi\left(e^{\frac{i}{\hbar}(H_s+H_R)t}e^{\frac{-i}{\hbar}(H_s+H_R)t}\right)H_{sR}e^{-\frac{i}{\hbar}(H_s+H_R)t} \\
&= -\frac{i}{\hbar}\left[\tilde{H}_{sR},\tilde{\chi}\right] \tag{2.5}
\end{aligned}
$$

noting that we have inserted 1 in the form of $e^{\frac{i}{\hbar}(H_s+H_R)t}e^{\frac{-i}{\hbar}(H_s+H_R)t}$ twice in the commutator so that we could define

$$\tilde{H}_{sR} \equiv e^{\frac{i}{\hbar}(H_s+H_R)t}H_{sR}e^{\frac{-i}{\hbar}(H_s+H_R)t} \tag{2.6}$$

an interaction picture coupling Hamiltonian term. After such a simplification there is time

dependence in both the state and the operators but we have eliminated the Hamiltonian terms for the system and the environment. We are left with a Schödinger equation in the interaction picture in terms of our state and only the coupling Hamiltonian, eq. (2.5).

2.2   Lindblad Master Equation

Now we need to integrate our state over time and trace over the reservoir to obtain an expression for $\dot{\tilde{\rho}}$

$$\int_0^t \frac{d}{dt}\tilde{\chi}dt' = -\frac{i}{\hbar}\int_0^t \left[\tilde{H}_{sR}(t'),\tilde{\chi}(t')\right]dt'$$

$$\tilde{\chi}(t) = \chi(0) - \frac{i}{\hbar}\int_0^t \left[\tilde{H}_{sR}(t'),\tilde{\chi}(t')\right]dt' \tag{2.7}$$

which will be our master equation in the interaction picture. Now plugging our expression for $\tilde{\chi}(t)$, eq. (2.7), back in to our expression for $\dot{\tilde{\chi}}$, eq. (2.5), we have

$$\dot{\tilde{\chi}}(t) = -\frac{i}{\hbar}\left[\tilde{H}_{sR}(t),\tilde{\chi}(0)\right] - \frac{1}{\hbar^2}\int_0^t \left[\tilde{H}_{sR}(t),\left[\tilde{H}_{sR}(t'),\tilde{\chi}(t')\right]\right]dt' \tag{2.8}$$

which is important to note is still an exact expression. We have done some convenient transformation to mathematically isolate our coupling Hamiltonian but this expression still fully describes our system.

The first assumption is made here when we assert that at our initial time there is no correlations between s and R thus $\chi(0) = \rho(0)R_0$,

$$\begin{aligned}
\text{Tr}_R \tilde{\chi}(t) &= \text{Tr}_R\left[e^{\frac{i}{\hbar}(H_s+H_R)t}\chi(t)e^{-\frac{i}{\hbar}(H_s+H_R)t}\right]\\
&= \text{Tr}_R\left[e^{\frac{i}{\hbar}H_st}e^{\frac{i}{\hbar}H_Rt}\chi(t)e^{-\frac{i}{\hbar}H_Rt}e^{-\frac{i}{\hbar}H_st}\right]\\
&= e^{\frac{i}{\hbar}H_st}\text{Tr}_R\left[e^{\frac{i}{\hbar}H_Rt}\chi(t)e^{-\frac{i}{\hbar}H_Rt}\right]e^{-\frac{i}{\hbar}H_st}\\
&= e^{\frac{i}{\hbar}H_st}\text{Tr}_R\left[\chi(t)e^{-\frac{i}{\hbar}H_Rt}e^{\frac{i}{\hbar}H_Rt}\right]e^{-\frac{i}{\hbar}H_st}\\
&= e^{\frac{i}{\hbar}H_st}\rho(t)e^{-\frac{i}{\hbar}H_st}\\
&= \tilde{\rho}(t)
\end{aligned} \tag{2.9}$$

allowing for a trace over our environment to leave us with the interaction density matrix for our system. If we now assume that the interaction starts at the initial time t, thus it can be said that $\chi(0) = \tilde{\chi}(0)$. To achieve an expression for our master equation we need to trace

on our state, eq. (2.8), over our environment,

$$\dot{\tilde{\rho}}(t) = \text{Tr}_R \dot{\tilde{\chi}}(t)$$

$$= -\frac{i}{\hbar} \text{Tr}_R \left[\tilde{H}_{sR}(t), \rho(0)R_0\right] - \frac{1}{\hbar^2} \int_0^t \text{Tr}_R \left[\tilde{H}_{sR}(t), \left[\tilde{H}_{sR}(t'), \tilde{\chi}(t')\right]\right] dt' \qquad (2.10)$$

noting that we have also plugged in the initial condition assumption. We now assume that the reservoir operators have zero mean the state R_0,

$$\text{Tr}_R \left[\tilde{H}_{sR}(t), \rho(0)R_0\right] = 0 \qquad (2.11)$$

which can generally arranged by how we build our Hamiltonian. This leaves us with a master equation in the form

$$\dot{\tilde{\rho}}(t) = -\frac{1}{\hbar^2} \int_0^t \text{Tr}_R \left[\tilde{H}_{sR}(t), \left[\tilde{H}_{sR}(t'), \tilde{\chi}(t')\right]\right] dt' \qquad (2.12)$$

with which we will now have to make some approximations to achieve the familiar Lindblad form.

2.3   Born-Markov Approximation

Now some approximations of our system will be made. First we assume that the coupling of s and R is very weak and that due to the large size of R, the system s has little affect on R. We will expand our state

$$\tilde{\chi}(t) = \tilde{\rho}(t)R_0 + O(H_{sR}) \qquad (2.13)$$

keeping only 2nd order terms in H_{sR}. We arrive at the form of the master equation that is known as the *Born Approximation*

$$\dot{\tilde{\rho}}(t) = -\frac{1}{\hbar^2} \int_0^t \text{Tr}_R \left[\tilde{H}_{sR}(t), \left[\tilde{H}_{sR}(t'), \tilde{\rho}(t')R_0\right]\right] dt' \qquad (2.14)$$

by plugging eq. (2.13) into the master equation, eq. (2.12).

Our last approximation has to do with the relative timescales of events. A state in such a system, with a bath like environment, quickly loses information but rarely gets it back. For example it is very likely that the atom will emit a photon into the surrounding space but it is very unlikely that the photon will ever make it back to the atom if it is in open space. It is thought to be approximately a one way process. A Markovian process is one

which only depends on the current state of the system and has no history. We achieve this approximation by letting $\rho(t') \to \rho(t)$ and acknowledging that our environment has no memory. This leads to the final form of our master equation known as the *Born-Markov Approximation*

$$\dot{\rho}(t) = -\frac{1}{\hbar^2} \int_0^t \text{Tr}_R \left[\tilde{H}_{sR}(t), \left[\tilde{H}_{sR}(t'), \tilde{\rho}(t) R_0 \right] \right] dt' \tag{2.15}$$

describing a quantum system weakly coupled to it's environment.

Lastly if the form of our interaction Hamiltonian can be written as a linear combination of system operators and environment operators $H_{sR} = \hbar \sum_i s_i \Gamma_i$ then we can make a final expansion to achieve a familiar form of the master equation

$$\dot{\rho}(t) = -\int_0^t \sum_{i,j} \text{Tr}_R \left[\tilde{s}_i(t)\tilde{\Gamma}_i(t), \left[\tilde{s}_j(t')\tilde{\Gamma}_j(t'), \tilde{\rho}(t) R_0 \right] \right] dt'$$

$$= -\int_0^t \sum_{i,j} dt' \{ [\tilde{s}_i(t)\tilde{s}_j(t')\tilde{\rho}(t) - \tilde{s}_j(t')\tilde{\rho}(t)\tilde{s}_i(t)] \langle \tilde{\Gamma}_i(t)\tilde{\Gamma}_j(t') \rangle$$

$$+ [\tilde{\rho}(t)\tilde{s}_j(t')\tilde{s}_i(t) - \tilde{s}_i(t)\tilde{\rho}(t)\tilde{s}_j(t')] \langle \tilde{\Gamma}_j(t')\tilde{\Gamma}_i(t) \rangle \} \tag{2.16}$$

noting here that the $\langle \tilde{\Gamma}_j(t')\tilde{\Gamma}_i(t) \rangle \propto \delta(t - t')$. The assumption that the environmental operators, $\tilde{\Gamma}$, effectively become deltas is because of the vast difference in time scales. The slow dynamics of the system and the fast decay of the reservoir correlation function justify this assumption. Thus we have finally derived the Lindblad super operator, $\mathcal{L}$, which describes a open dissipative quantum system. In the interaction picture we can define the Lindblad super operator which acts on a density matrix ρ such that

$$\mathcal{L}(\hat{O})\rho = \hat{O}\rho\hat{O}^\dagger - \frac{1}{2} \left(\hat{O}^\dagger \hat{O}\rho + \rho\hat{O}^\dagger\hat{O} \right) \tag{2.17}$$

where $\hat{O}$ is the collapse operator. Before we continue we will redefine $\mathcal{L}$ to it's most general form in the Schrödinger picture

$$\dot{\rho} = \mathcal{L}\rho \tag{2.18}$$

with

$$\mathcal{L} = \frac{1}{i\hbar}[H_s, \cdot] + \sum_j \frac{\gamma_j}{2} (2\hat{O}_j \cdot \hat{O}_j^\dagger - \hat{O}_j^\dagger \hat{O}_j \cdot - \cdot \hat{O}_j^\dagger \hat{O}_j) \tag{2.19}$$

noting the reintroduction of the Hamiltonian which was mathematically eliminated in the interaction picture derivation of the Schrödinger equation, eq. (2.5). It is this form of the master equation which will be used to describe our collection of Rydberg atoms.

2.4 Quantum Trajectories

Our treatment of the master equation in the previous section is a valid approach to describing our open quantum systems, but ultimately they are ensemble-averaged quantities. If our goal is to study quantum stochastic processes then we need a new approach, which is outlined by Carmichael [7], known as Quantum Trajectories. The goal of such a model, as with most of quantum optics, is to describe the interaction of light and matter focusing specifically on the quantum dynamics. In an ideal system where all of the scattered photons can be recovered one can define the density matrix in an interesting way. If our system is written in the familiar Lindblad form, eq. (2.18), then we need to define our state at a sum over all possible scattering event. This is an attempt to *unravel* the density operator as

$$\rho(t) = \sum_{\text{REC}} P(\text{REC}) \, |\psi_{\text{REC}}(t)\rangle \, \langle\psi_{\text{REC}}(t)| \tag{2.20}$$

where $|\psi_{\text{REC}}(t)\rangle$ is a *conditional state* of a single scattering event in the system s. $P(\text{REC})$ is the probability for a specific scattering even to occur. It is important to note that this is a full record of the specific sequence of scattering events such that

$$\sum_{\text{REC}} P(\text{REC}) = 1 \tag{2.21}$$

leaving all scattering events possible.

To describe a stochastic system that exhibits quantum jumps similar to the ones Bohr and Einstein attempted to model we must consider a Monte Carlo approach to our system. First let's introduce the algorithm and then discuss the important components.

Quantum Trajectory Algorithm (for a two state system)

> **Calculate** jump probability: $p_\downarrow = \sum_i \gamma_i \langle O_i^\dagger O_i \rangle \Delta t$
> **Generate** a uniformly distributed random number, r_k such that $0 < r_k \leq 1$
> **if** $p > r_k$ **then**
> > **Pick** Jump based on a weighted probability of $p_i = \gamma_i \langle O_i^\dagger O_i \rangle \Delta t$
> > **Apply** Jump: $|\psi\rangle \rightarrow O_i |\psi\rangle$
> **else**
> > **Evolve** with non-Hermitian Hamiltonian: $|\psi\rangle \rightarrow |\psi\rangle - \frac{i}{\hbar} H_B |\psi\rangle \Delta t$

where $p_\downarrow$ is the total probability for a jump to occur and p_i is the probability for the ith collapse operator to be applied. O_i are the collapse operators associated with the system.

It's important to note that there can be many jump operators for a system but one is needed. H_B is a non-Hermitian Hamiltonian defined as

$$H_B = H - i\hbar \sum_i \frac{\gamma_i}{2} O_i^\dagger O_i \tag{2.22}$$

that is needed to model the results of a null measurement. If jump doesn't occur over a period of time the probability that the atom is in the excited state decreases. This is a result of the Bayesian statistics used to describe our system. The quantum trajectory algorithm is general in the sense that it can be used to describe the evolution of a state that is starting in a specific quantum state $|1\rangle$, $|2\rangle$ or a superposition of $(|1\rangle + |2\rangle)$ without knowledge of the initial state.

Given these considerations the system will exhibit similar stochastic processes which we are attempting to model. After significant time evolution we will generate a record of scattering events

$$\text{REC} \equiv \left\{ \begin{matrix} \cdots \\ \cdots \end{matrix}, \emptyset, \frac{\gamma_{\downarrow j}}{T_{k-1}}, \emptyset, \frac{\gamma_{\downarrow j}}{T_k}, \emptyset, \frac{\gamma_{\downarrow j}}{T_{k+1}}, \emptyset, \frac{\gamma_{\downarrow j}}{T_{k+2}}, \emptyset, \begin{matrix} \cdots \\ \cdots \end{matrix} \right\} \tag{2.23}$$

where $\emptyset$ represents regular time evolution, $\gamma_{\downarrow j}$ represents the jth collapse operator being applied and T_K is the time when that jump occurs. It is this method which is used to study the collective quantum jump of Rydberg atoms in my simulations.

Chapter 3

Collective Rydberg Systems

Many recent publications have highlighted the experimental methods of trapping and interacting with small collections of Rydberg atoms [11, 12]. These advancements motivate the study of many body Rydberg systems. For our models we will be exploring the collective nature of these systems and interesting phenomena that emerges.

3.1 Hamiltonian

For our model we will consider a system of N driven, damped, two-level Rydberg atoms in various geometric configurations. Making the electric dipole and rotating wave approximations the Hamiltonian in the interaction picture is ($\hbar = 1$)

$$H = \sum_{j=1}^{N} \left[-\Delta |e\rangle\langle e|_j + \frac{\Omega}{2} \left(\sigma_{j+} + \sigma_{j-} \right) \right] + \frac{V}{N-1} \sum_{j<k=1}^{N} |e\rangle\langle e|_j \otimes |e\rangle\langle e|_k \qquad (3.1)$$

where σ_{j-} is the Pauli lowering operator for the jth atom, $\Delta = \omega_\ell - \omega_0$ is the detuning of the driving laser from the atomic resonance, Ω is the Rabi frequency, and V is a constant that accounts for the long range interaction between the Rydberg atoms in their excited states. Note that the term involving the Rydberg interaction is only summed for each pair of atoms and thus the limits are $j < k = 1$.

An interesting phenomenon is the possibility of a two photon excitation (fig. 3.1a). When the first atom is excited to a Rydberg state, the surrounding atoms experience a shift in their respective energy levels due to the van Der Waals interaction. This energy shift would introduce a Rydberg blockade which would suppress all excitations after the first atom is excited. This is a familiar behavior that has been studied at length for cold Rydberg atoms [13]. Now if we run our simulation for a wide enough range of detunings from the atomic

(a) Two photon excitation
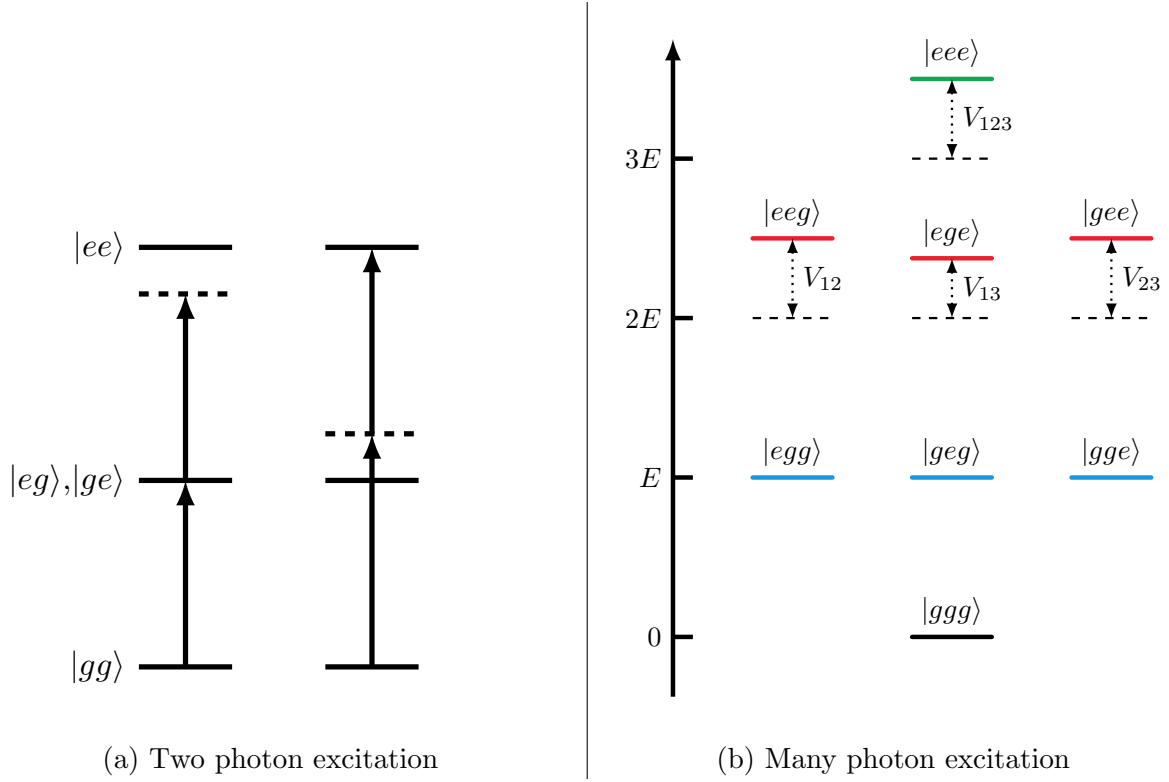
(b) Many photon excitation

Figure 3.1: Energy level diagrams for Rydberg systems. (a) Shows how a two atom system is more likely to make a two photon excitation when driven off resonance. (b) Shows the more complicated diagram for three atoms arranged in a line.

resonance, Δ, there will be an increased probability of a two photon excitation for specific detunings. This makes the two photon excitation more favorable than others and is a place where collective jumps are more likely to occur. This scenario was observed for two atoms when our detuning was half of our Rydberg shift, $\Delta = V/2$. For many atoms there are more possibilities for more atoms to be excited in these type of multiphoton excitations. The defining characteristic of this system is the relative distance each atom is from it's nearest neighbor. The further the atoms are away from each other the less of a shift will occur and thus the Rydberg effects will be reduced. Creating an energy level diagram for a line of three atoms (fig. 3.1b) becomes more complicated due to the many neighbors that have varying affects. Notice that the shift between the first and the second atoms and the shift between the second and the third atom are equal but the first and third Rydberg shift is reduced because of the increased distance.

3.2 Collapse Operators

We include damping in our system using the familiar Linbald form of the master equation derived in the Born-Markov approximation eq. (2.18). For independent spontaneous emission we use the collapse operator σ_{j-} thus we have:

$$\dot{\rho} = -i\,[H, \rho] + \frac{\gamma}{2}\sum_{j=1}^{N}\left(2\sigma_{j-}\rho\sigma_{j+} - \rho\sigma_{j+}\sigma_{j-} - \sigma_{j+}\sigma_{j-}\rho\right) \tag{3.2}$$

Note that each atom has its own individual collapse operator. For collective spontaneous emission a collective collapse operator was utilized where $J_- = \sum_{j=1}^{N}\sigma_{j-}$ to allow for fully collective emission.

$$\dot{\rho} = -i\,[H, \rho] + \frac{\gamma}{2}\left(2J_-\rho J_+ - \rho J_+ J_- - J_+ J_-\rho\right) \tag{3.3}$$

An intermediate option was also explored where the system could be varied between fully collective and fully independent spontaneous emission. This was accomplished by introducing a γ_{ij} term that was used to define the spontaneous emission of a set of dipoles.[14, 15]

$$\gamma_{ij} = \gamma\frac{3}{2}\left\{[1 - (\hat{d}\cdot\hat{r}_{ij})^2]\frac{\sin\xi_{ij}}{\xi_{ij}} + [1 - 3(\hat{d}\cdot\hat{r}_{ij})^2]\left(\frac{\cos\xi_{ij}}{\xi_{ij}^2} - \frac{\sin\xi_{ij}}{\xi_{ij}^3}\right)\right\} \tag{3.4}$$

where

$$\xi_{ij} \equiv k_0 r_{ij} = 2\pi r_{ij}/\lambda_0, \qquad \mathbf{r}_{ij} = r_{ij}\hat{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j \tag{3.5}$$

We only looked at cases where all of the dipoles were arranged in a plane and thus $(\hat{d}\cdot\hat{r}_{ij}) = 0$ simplifying our calculation. It is important to note that there is an oscillatory nature to the γ_{ij} matrix as a function of R and this could have interesting effects on our system. The eigenvalues, λ_i, and eigenvectors, $\vec{v}_i$, where calculated for the γ_{ij} matrix and used to determine the collapse operators for our system by

$$J_i = \sqrt{\lambda_i}\sum_{j}^{N}\vec{v}_{ij}\sigma_{j-} \tag{3.6}$$

where J_i is the ith collapse operator, similar to σ_{j-} and not to be confused with the collective operator J_-. It was important to check the behavior of this equation at limiting cases to make sure it would approach independent and collective emission for large and small separation distances, respectively (fig. 3.2). For a system that is separated by a large distance, $R \gg 1$, the off diagonal terms of our γ_{ij} matrix approach zero creating matrix similar to the identity matrix. Matrices of this form have eigenvalues that are all 1 and eigenvectors with only a

$$\text{Independent case:} \qquad \begin{pmatrix} 1 & 0 & \cdots \\ 0 & 1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \implies \text{all } \lambda_i \to 1$$

$$\text{Collective case:} \qquad \begin{pmatrix} 1 & 1 & \cdots \\ 1 & 1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \implies \text{one } \lambda \to N$$

Figure 3.2: The coupling matrix γ_{ij} takes on different forms for the limits of the separation distance. For $R \gg 1$ the system is independent and for $R \ll 1$ the system is collective.

single nonzero value. Using eq. (3.6) to build the collapse operators the system mimics the behavior of the independent spontaneous emission case.

At the other limit, $R \ll 1$, where our distances become very small the off diagonal terms of the γ_{ij} matrix would all approach 1 creating a matrix composed entirely of ones, sometimes referred to as the all-ones matrix. Matrices of this from have only a single nonzero eigenvalue that approaches the value of N. This creates a jump operator with a value that when multiplied by our normalized eigenvectors leaves us with just a sum of σ_j- which is the correct collapse operator for collective spontaneous emission. Using this method we can explore the dynamics of our system as it moves from an independent regime to a collective regime.

3.3   Geometry

We ran our simulation for a range of detunings to generate a probe spectrum for a line, triangle and square configurations (fig. 3.3).The Rydberg interaction, V, was changed from a originally being a constant for all atoms to a function of distance. The Euclidean distance was calculated for all pairs of atoms and then the amount the energies were shifted was calculated with appropriate scaling. Both a $1/r^6$ and a $1/r^3$ case was used to define the interaction range and the Rydberg interaction was multiplied by the scale factor. For each configuration the separation distance was also varied to study how the changing Rydberg shifts affected the probe spectrum. Making this change required that each interaction be defined in a term V_{ij} rather than having a single V term define the total interaction for all atoms. By changing this interaction behavior the geometry and characteristic separation distance of the system becomes an important factor. The number of nearest neighbors
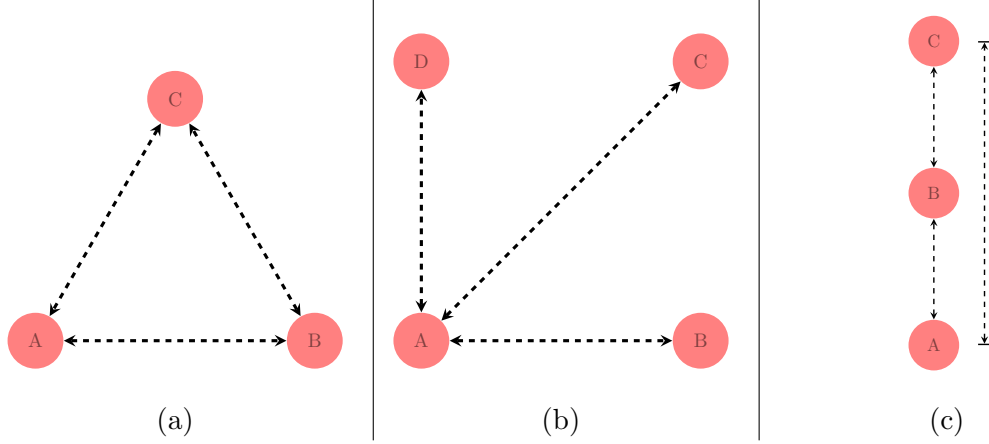
Figure 3.3: Atomic configurations for a triangle, square, and a line of atoms. The geometry of the system affects the magnitude of the Rydberg shift and is important to consider.

interactions adds to the magnitude of the shift in energy levels. I also calculated some atom-atom cross correlations, $g_{ij}^{(2)}(0)$, and stochastic collective quantum jumps for independent, collective, and intermediate spontaneous emission.

The geometry of the system defines how the energy levels shift. For example an equilateral triangle (fig. 3.3a) will have constant V because all of the atoms are equally spaced. In fact this is the only geometry for $N > 2$ for which this is the case. A square will not have interactions of all the same magnitude with the (fig. 3.3b). Only the two nearest neighbors will contribute similar shifts. The energy shift for a line of three atoms (fig. 3.3c) would be the same for the first and second atom as well as the second and third atom but less for the first and third. Taking into account these nearest neighbors interactions helps us to better understand the system.

Chapter 4

Quantum Toolbox in Python

For this project the Quantum Toolbox in Python (QuTiP) was used heavily [16] . QuTiP
has defined a class for quantum objects (`Qobj`) where they can be manipulated with familiar
quantum routines such as `.dag()` for taking the dagger of a state or operator and `.unit()`
for normalization. The `Qobj` also contains important information such as whether they are
Hermitian and shape to ensure consistent quantum simulations. There are many familiar
quantum objects built into QuTiP , such as the Pauli matrices (fig. 4.1), which were used
to build our collective collapse operator. Utilizing this framework, performing quantum
simulations becomes very similar to the way you would write out the mathematics and
minimizes the learning curve.

```
>>> qutip.sigmax()
Quantum object: dims = [[2], [2]], shape = [2, 2], type = oper, isherm = True
Qobj data =
[[ 0.  1.]
 [ 1.  0.]]
```

Figure 4.1: Shown above is an example of the basic output for a printed `Qobj` in this case
the Pauli spin operator σ_x

4.1   Time Evolution

There are built in algorithms for time evolution; specifically the master equation, `mesolve`,
and the Monte Carlo quantum trajectory function, `mcsolve`, were used to study the dynamics
of our system. The `mesolve` function takes in a Hamiltonian, an initial state and a list of time
you want your system to evolve over. If you want to add dissipation to your system there is a
parameter for the collapse operators which is what we changed to replicate independent and

collective emission. Expectation values can also be calculated for each time step through the expectation parameter. The `mcsolve` function takes similar parameters but also will let you control the number of trajectories made. In our case we followed a single trajectory over many lifetimes to see the bistability. QuTiP returns an object called `Results` which contains lots of important information. If no expectation values were requested then you will receive an array of your state at every time step. If you did provide an expectation value to calculate you will receive a value for every time step. If you are performing `mcsolve` you will also receive information about when and which collapse operator was used for every trajectory. This information was useful in exploring the dynamics of our system as we could see that many successive applications of our collapse operators lead to a jump event.

QuTiP has many other features for visualization and quantum simulations. Included on the website are many tutorials which detail how to perform many calculations. QuTiP is still under active development and released QuTiP 3.1.0 on Dec 31, 2014.

Chapter 5

Results

5.1   Probe Spectrum

The preliminary form of analysis was to calculate a probe spectrum of our system in a steady state for a wide range of detunings. The expectation value $\langle \sigma_{i+} \sigma_{i-} \rangle$ was calculated as a function of detuning of the laser to explore where multiphoton excitations were more likely to occur when our system was being driven off resonance. Probe spectra were made for various parameters including independent and collective spontaneous emission types (Fig. 5.1).
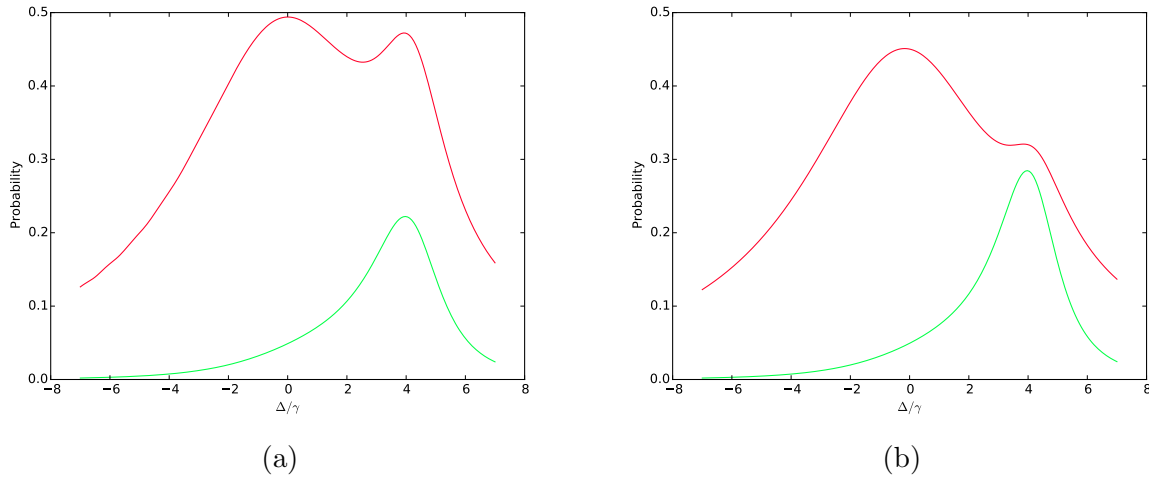
Figure 5.1: We plot the probability to excite 1 or 2 atoms for $N = 2$, $\gamma = 1$, $V = 8$, and $\Omega = 4$ for (a) independent spontaneous emission and (b) collective spontaneous emission

The bump seen at $\Delta/\gamma = 4$ is a result of a two photon excitation discussed earlier due to the Rydberg blockade. Upon comparison of fig. 5.1a and fig. 5.1b, there is some insight into why different emission type changes the excitation probability. The single photon excitation was reduced by collective emission but the two photon behavior was enhanced.

The probability to have one atom in the excited state is reduced when the emission type is made collective because it is now emitting twice as fast due to the collective nature of the emission. The atoms also absorbs light and become excited twice as fast because the driving is always collective. If both atoms are excited and then one of them emits collectively it becomes an entangled state which has enhanced emission and absorption. For independent emission the single atom excited state following emission does not have enhanced emission and absorption. Therefore probability that both atoms are excited is enhanced when the emission is made collective and the probability of single atom excitation is inhibited.
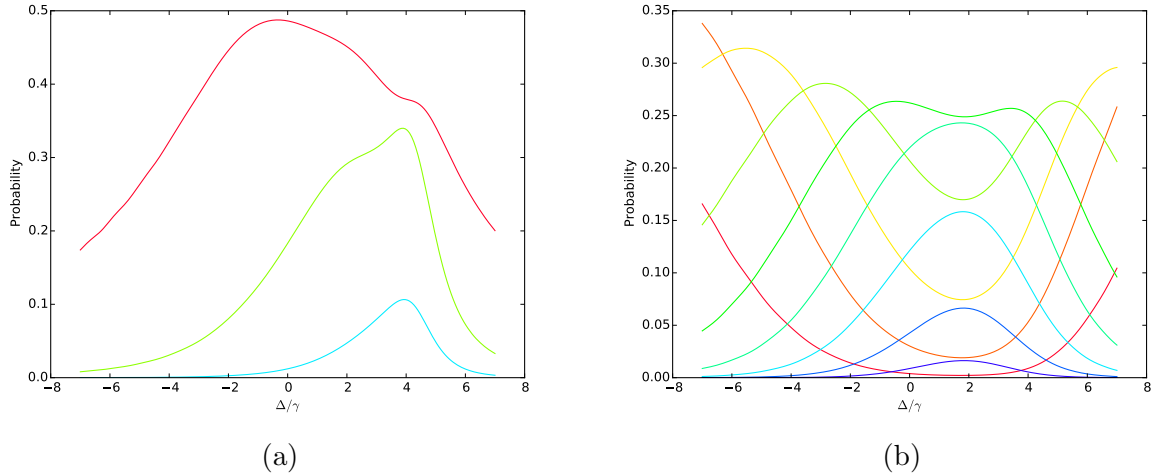

(a)   (b)

Figure 5.2: In both figures the parameters are $\gamma = 1$, $V = 8$, and $\Omega = 4$ with N changing based on the shape. (a) A probe spectrum of a triangle of atoms with 1, 2, or 3 atoms excited. A probe spectrum of a 3x3 square of atoms with 1, 2, 3, 4, 5, 6, 7, 8, or 9 atoms excited.

Further studies of the excitation probabilities were explored by increasing N, the numbers of atoms in the system. The energy shift associated with the dipole-dipole interaction has a strong dependence on distance so various geometries of atoms were studied (Fig. 5.2). A variable energy shift was introduced into the Hamiltonian, V_{ij} for all of the pairs of atoms. Probe spectra were made for shapes such as a line, triangle, square, and a larger 3 by 3 square of atoms. The probe spectra of the more complicated geometries were harder to directly analyze but did provide a good starting place to understand how the system responds to various detunings as well as a changing distance. This method could be further expanded into three dimensions but careful consideration must be taken with respect to how the intermediate emission collapse operators are defined , eq. (3.4), because $(\hat{d} \cdot \hat{r}_{ij}) \neq 0$. The distances were varied to see for what distances the Rydberg interaction dominated. Animations were made showing how the system evolves as a function of distance. The

19

energy required to excite multiple atoms was increased due to the larger Rydberg effect for short distances. This behavior lead to interesting dynamics when animated causing the peaks to shift to the right and spread further apart as separation distance decreased.

5.2 Cross Correlations

We calculated the normalized atom-atom intensity cross correlation

$$g_{ij}^{(2)}(0) = \frac{\langle \sigma_{i+}\sigma_{i-}\sigma_{j+}\sigma_{j-}\rangle}{\langle \sigma_{i+}\sigma_{i-}\rangle\langle \sigma_{j+}\sigma_{j-}\rangle}. \tag{5.1}$$

Note that for both collective and independent emission the master equation is symmetric with respect to exchange of atom labels so it does not matter which pair of atoms we use. For our adaptive spontaneous emission case where the emission type can vary this symmetry is not true.

We characterized the intensity fluctuations by plotting the atom-atom cross correlation $g_{ij}^{(2)}(0)$ in fig. 5.3. A value of $g_{ij}^{(2)}(0) = 1$ would correspond system of discrete events happening completely uncorrelated, exhibiting shot noise. For values of $g_{ij}^{(2)}(0) > 1$ events would be bunched and we would expect to see more noise in our system. Therefore we were looking for a peak in the $g_{ij}^{(2)}(0)$ plot representing a stronger possibility of collective jumps.

(a)   (b)
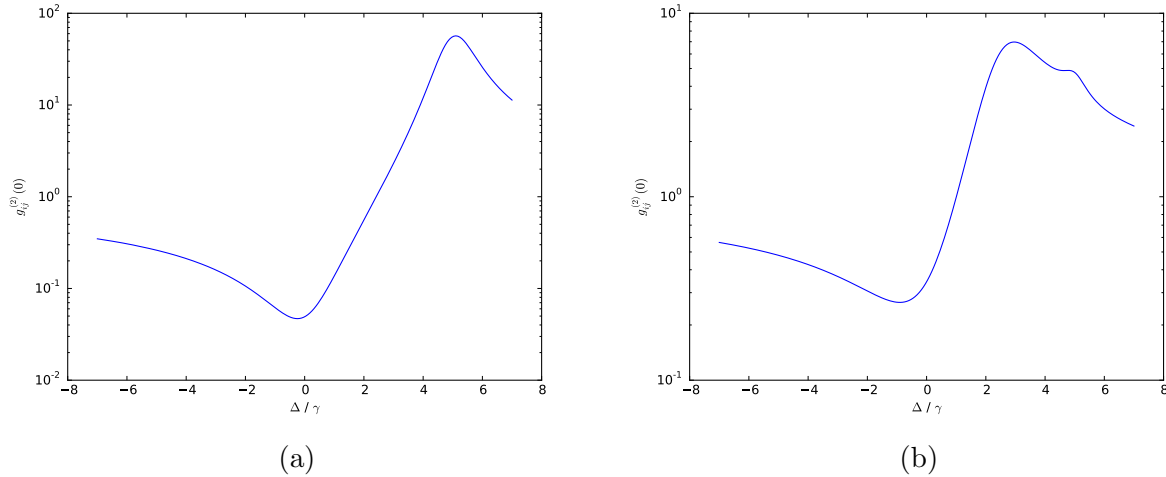
Figure 5.3: We plot the atom-atom cross correlation for independently emitting groups of atoms. (a) $g_{ij}^{(2)}(0)$ for just two atoms. (b) $g_{ij}^{(2)}(0)$ for a triangle of atoms. The bump seen for the triangle case is where a three photon resonance would be expected.

These correlations can be predicted by analysing the Rydberg term of our Hamiltonian. Let's examine at the case of an equilateral triangle of atoms . The leading term $V/(N-1)$

would be the same for a two photon or three photo excitation, in this case 5. Now we need to multiply by the number of pairs of excited atoms and then divide by the number of photons needed to reach that state. For the case of the two photon excitation there is only one pair of excited atoms and it takes 2 photons to reach the state, so we would expect a bump at $5/2 = 2.5$. For the three photon excitation there are three pairs of excited atoms and it would take a total of three atoms to make the excitation so we would also expect a bump a $5(3/3) = 5$. Both of these results are seen in the cross correlation plot for the triangle configuration (fig. 5.3b). These cross correlation plots can be more revealing of the nature of the system when more atoms are introduced.

5.3 Collective Quantum Jumps

To have these Rydberg atoms exhibit collective quantum jumps an even larger number of atoms was considered as the effect scales with N. To explore the state of a single quantum trajectory a Monte Carlo method was implemented in QuTiP (`mcsolve`). The excitation population of the system was computed over many life times. The simulation was run for a wide range of separation lengths from $R = 100$ to $R = 0.1$ (fig. 5.4) where R is a fraction of the emission wavelength, λ.

(a)   (b)
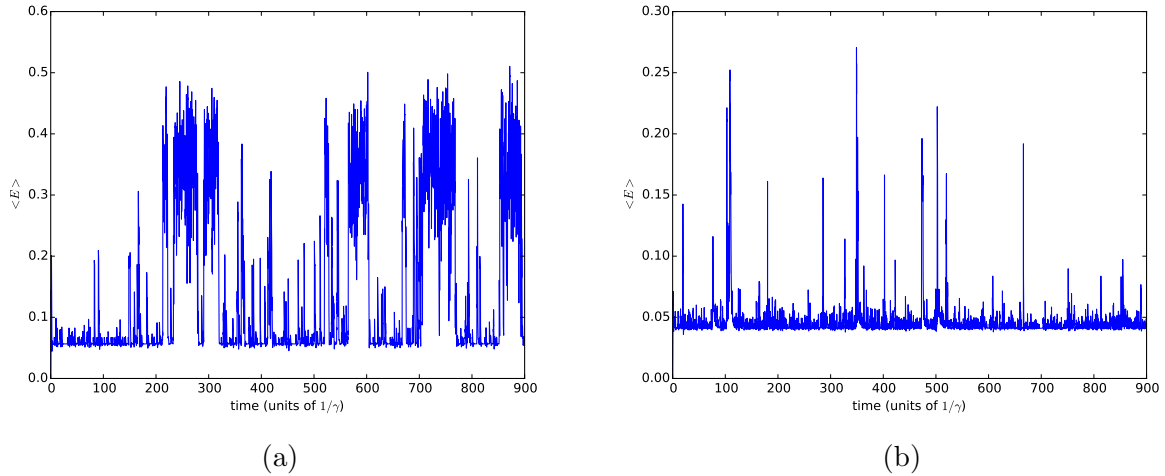
Figure 5.4: Collective jumps between a low excitation "dark" state and a high excitation "bright" state, for $N = 16, \Delta = 3.5, \Omega = 1.5, V = 8$. Note $\langle E \rangle \equiv \langle \sigma_+ \sigma_- \rangle$. (a) separation distance is $R = 0.5$ and (b) separation distance is $R = 0.115$, where R is a fraction of the emission wavelength, λ

As the atoms get closer together their emission type becomes more like collective spontaneous emission and the quantum jumps seem to be diminished. The size of the fluctuations

are on a similar order as the normal background and no collective jumps to a bright state seem to have occurred. Either the jumps maybe be more spread out over many lifetimes or the system is in the bright state for a much shorter time.

Jump statistics were studied to characterized the disappearance of the collective jumps. A single quantum trajectory was run for $10,000/\gamma$ so that many jump events could be studied. The expectation value of the excitation population $\langle \sigma_+ \sigma_- \rangle$, which we defined as $\langle E \rangle$, was calculated to determine if our system was in a "bright" or "dark" state. A jump event was characterized by choosing an arbitrary cutoff value and counting when our expectation value crossed this point. In our case the value 0.2 was used to calculate jump statistics. For a typical trajectory there were ~400 jump events which could be varied by performing longer runs. The jump length was calculated for each jump event to characterize how long the system stayed in the bright state.

(a)   (b)
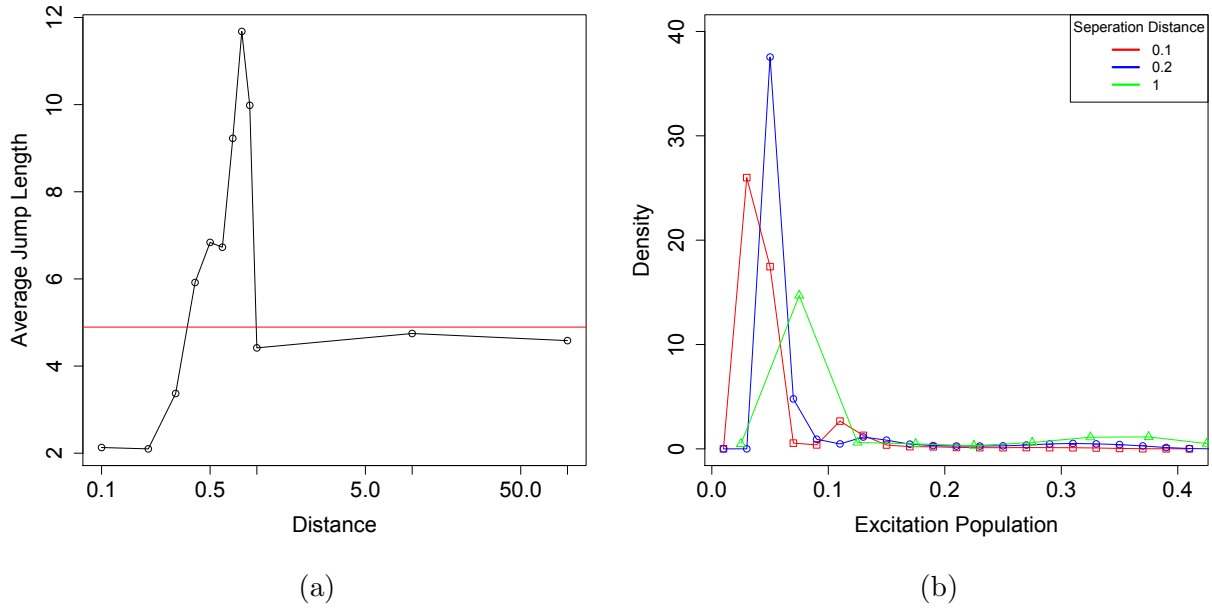
Figure 5.5: (a) Average jump length for a wide range of separation distances. Note that the average jump length approaches the independent jump length for large values of R. (b) Average excitation population density shows the optical bistability. The parameters for all of the runs are: $N = 16, \Delta = 3.5, \Omega = 1.5, V = 10$

Average jump length was calculated to show that the bistablity is inhibited for short separation distanced and that for large separation distances is similar to the independent case (fig. 5.5a). An interesting phenomenon is that for distances $0.3 < r < 1.0$ the jump length is enhanced which is an unexpected result. A histogram of the excitation population was also made to better quantify the optical bistability (fig. 5.5b). Notice that for larger separation

distances the population density is shared between two peaks. There is a common strong peak at around $\langle E \rangle = 0.05$ but for larger separation some of the distances the population density is moved to a peak around $\langle E \rangle = 0.35$. These correspond well to the "bright" and "dark" states observed in fig. 5.4.

(a)

(b)

(c)

(d)
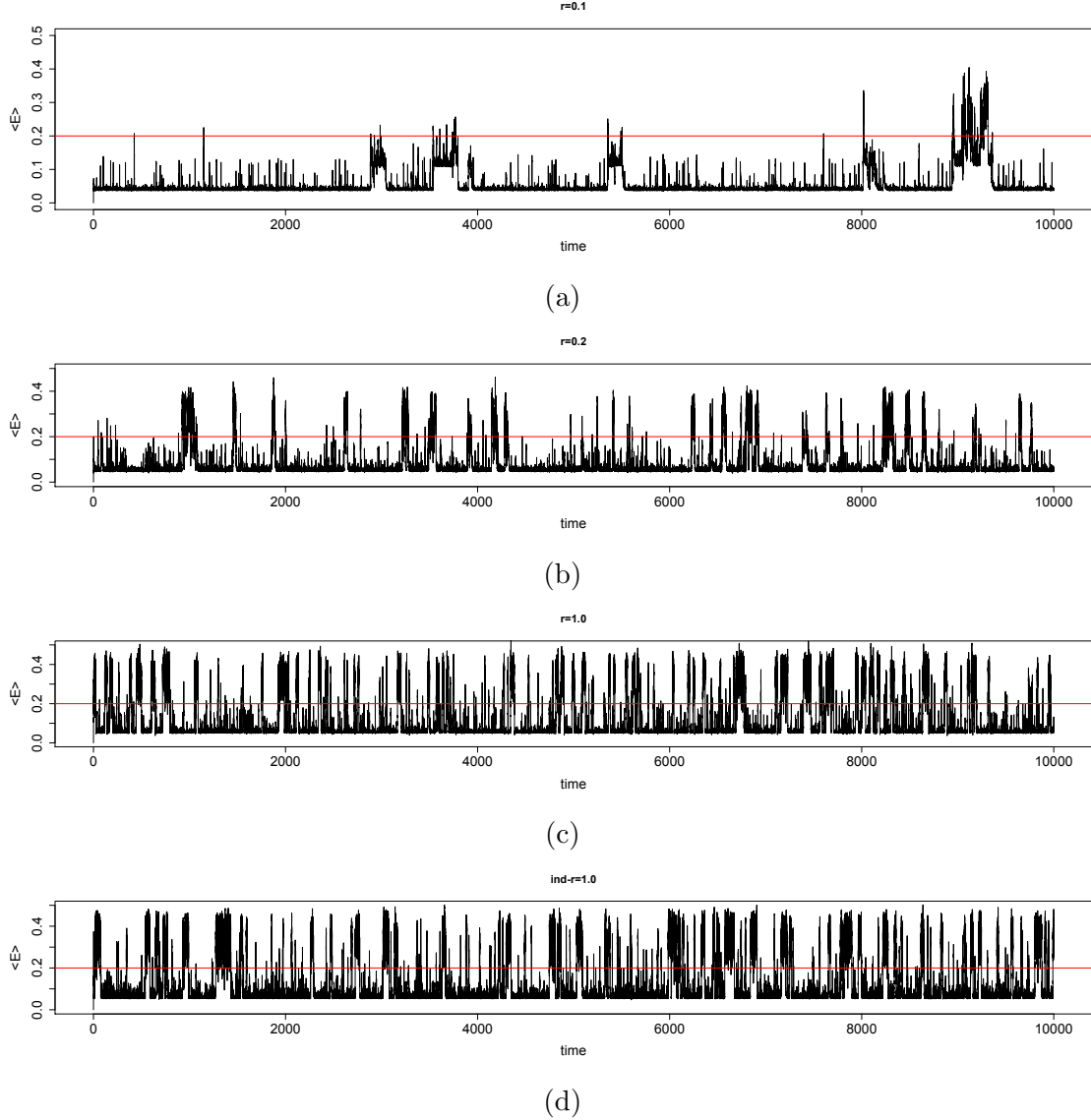
Figure 5.6: Excitation population for 16 atoms for various separation distances. (a) $r = 0.1$, (b) $r = 0.2$, (c) $r = 1.0$, (d) $r = 1.0$ but it is the fully independent emission case. Notice how for small R the jumps are greatly diminished. Parameters used $N = 16, \Delta = 3.5, \Omega = 1.5, V = 10$

The longer $10,000\,\gamma$ runs (fig. 5.6) are shown to be inhibited for short separation distances. This shows that the intermediate emission model correctly interpolates between the fully independent and collective regime. Note that the fully independent case (fig. 5.6d) is very

similar to the intermediate case separation distance of $R = 1.0$ (fig. 5.6c) but as the distance is shortened the bistability becomes weaker. At $R = 0.2$ (fig. 5.6b) there are still some short jumps that almost disappears at $R = 0.1$ (fig. 5.6a) which is what is expected for the fully collective regime.

5.4   Optical Bloch Solutions

An explanation for the bistability comes from analysing the optical Bloch equations for a system of two level atoms. One can show the dynamics for such a system are

$$\dot{\bar{\rho}}_{ee} = -\Omega \operatorname{Im} \bar{\rho}_{eg} - \gamma \bar{\rho}_{ee} \tag{5.2}$$

$$\dot{\bar{\rho}}_{eg} = i \left(\Delta - V \bar{\rho}_{ee} \right) \bar{\rho}_{eg} - \frac{\gamma}{2} \bar{\rho}_{eg} + i\Omega \left(\bar{\rho}_{ee} - \frac{1}{2} \right) \tag{5.3}$$

under a mean-field theory approach. It is important to note that the mean-field theory approach assumes that the density matrix factorizes by atom,

$$\rho = \bigotimes_{j=1}^{N} \rho_j \tag{5.4}$$

and therefore there are no correlations between the atoms. When we attempt to describe our system with collective emission under the same framework this assumption breaks down. We will first look at the steady state for the independent emission case. Setting eq. (5.2) equal to zero we find

$$\gamma \bar{\rho}_{ee} = -\Omega \operatorname{Im} \bar{\rho}_{eg} \tag{5.5}$$

and plugging eq. (5.5) into eq. (5.3) gives

$$0 = i \left(\Delta + V \frac{\Omega}{\gamma} \operatorname{Im} \bar{\rho}_{eg} \right) \bar{\rho}_{eg} - \frac{\gamma}{2} \bar{\rho}_{eg} + i\Omega \left(-\frac{\Omega}{\gamma} \operatorname{Im} \bar{\rho}_{eg} - \frac{1}{2} \right) \tag{5.6}$$

to be our steady state equation. We now will assume that $\bar{\rho}_{eg}$ is a complex number of the form $\bar{\rho}_{eg} = a + ib$ and plug that into eq. (5.6),

$$0 = i \left(\Delta + V \frac{\Omega}{\gamma} b \right) (a + ib) - \frac{\gamma}{2} (a + ib) + i\Omega \left(-\frac{\Omega}{\gamma} b - \frac{1}{2} \right)$$

$$0 = i \left(\Delta + V \frac{\Omega}{\gamma} b \right) a - \left(\Delta + V \frac{\Omega}{\gamma} b \right) b - \frac{\gamma}{2} a - i \frac{\gamma}{2} b + i\Omega \left(-\frac{\Omega}{\gamma} b - \frac{1}{2} \right) \tag{5.7}$$

giving an expression for the steady state of $\bar{\rho}_{eg}$. By gathering the appropriate terms,

$$0 = -\left(\Delta + V\frac{\Omega}{\gamma}b\right)b - \frac{\gamma}{2}a \tag{5.8}$$

$$0 = \left(\Delta + V\frac{\Omega}{\gamma}b\right)a - \frac{\gamma}{2}b + \Omega\left(-\frac{\Omega}{\gamma}b - \frac{1}{2}\right) \tag{5.9}$$

we can split up $\bar{\rho}_{eg}$ into real and imaginary parts. We can now solve eq. (5.8) for a and plug it into eq. (5.9)

$$a = -\frac{2}{\gamma}\left(\Delta + V\frac{\Omega}{\gamma}b\right)b \tag{5.10}$$

$$0 = -\frac{2}{\gamma}\left(\Delta + V\frac{\Omega}{\gamma}b\right)^2 b - \frac{\gamma}{2}b + \Omega\left(-\frac{\Omega}{\gamma}b - \frac{1}{2}\right) \tag{5.11}$$

eliminating a from our solution. We already have an expression, eq. (5.5), for which we can replace b with

$$\gamma\bar{\rho}_{ee} = -\Omega b$$

$$b = -\frac{\gamma}{\Omega}\bar{\rho}_{ee} \tag{5.12}$$

from the optical Bloch equations discussed earlier. Plugging our expression for b, eq. (5.12), into eq. (5.11) we find

$$0 = -\frac{2}{\gamma}\left(\Delta + V\frac{\Omega}{\gamma}\left(-\frac{\gamma}{\Omega}\bar{\rho}_{ee}\right)\right)^2\left(-\frac{\gamma}{\Omega}\bar{\rho}_{ee}\right) - \frac{\gamma}{2}\left(-\frac{\gamma}{\Omega}\bar{\rho}_{ee}\right) + \Omega\left(-\frac{\Omega}{\gamma}\left(-\frac{\gamma}{\Omega}\bar{\rho}_{ee}\right) - \frac{1}{2}\right)$$

$$0 = \frac{2}{\Omega}\left(\Delta - V\bar{\rho}_{ee}\right)^2\bar{\rho}_{ee} + \frac{\gamma^2}{2\Omega}\bar{\rho}_{ee} + \Omega\left(\bar{\rho}_{ee} - \frac{1}{2}\right) \tag{5.13}$$

which represents the solutions to our steady state system. We now need to solve eq. (5.13) in order to understand the stability of our system. Mathematica was utilized to solve eq. (5.13) which showed showed multiple stable points in our system (fig. 5.7). Typically there will be one stable point and two unstable points leading to a stable steady state solution being one of the tails of the graphs. Collective jumps occure when there are two stable solutions and one unstable solution caused by driving off resonance. It is this region of the detuning which are interested in. Plots of the solutions, such as fig. 5.7a, were made in Mathematica utilizing the `Manipulate` function allowing us to easily see how different parameters affected the bistability.
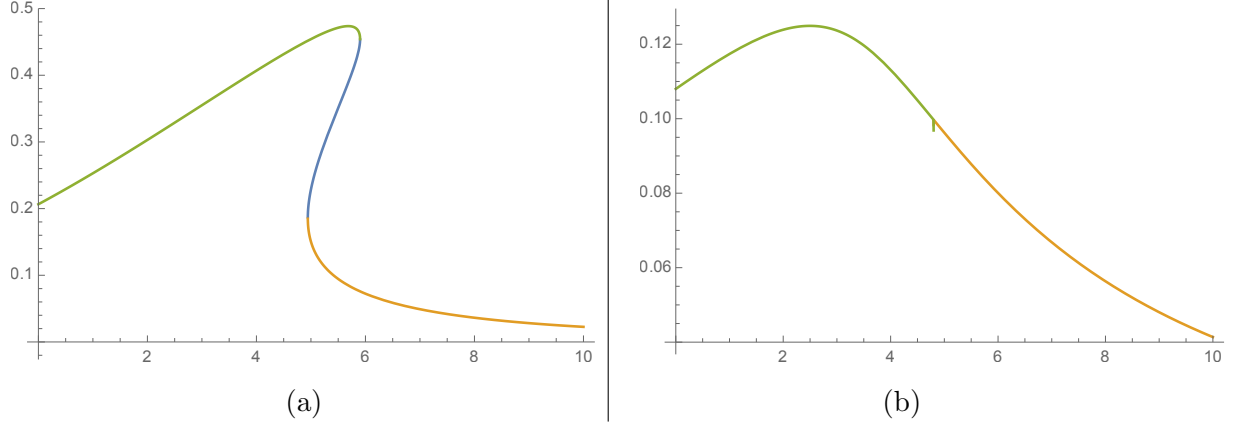
Figure 5.7: (a) The 1st, 2nd, and 3rd solution for the independent case for $V = 12, \Omega = 3, \gamma = 1$ and (b) the collective case for $V = 20, \Omega = 5, \gamma = 1, N = 16$

For collective emission we utilized a counting argument to understand how adding more atoms to the system affected the optical Bloch equations. Following a similar argument to the derivation of the independent steady state solutions, eq. (5.13), we find for collective emission the steady state is represented by solutions to

$$0 = \left(\bar{\rho}_{ee} - \frac{1}{2}\right) + \frac{2}{\Omega^2}\bar{\rho}_{ee}\left[(\Delta - V\bar{\rho}_{ee})^2 + \frac{\gamma^2}{4}(2(N-1)\bar{\rho}_{ee} - N)^2\right] \tag{5.14}$$

noting that eq. (5.14) is of the same form as eq. (5.13) with a correction of the spontaneous emission term, $\gamma \to \gamma(2(N-1)\bar{\rho}_{ee} - N)$, which introduced the N dependence on our system.

Studying the behavior of the collective emission case showed that the bistability should go aways for a system of 16 atoms which was expected. We then constructed a situation in which the bistability would still occur for given eq. (5.14), and ran quantum trajectories at those detunings. The expected result was that the jumps would return but the jumps were still missing. This put in question the validity of the mean-field theory when modeling collective systems. The major assumption that the density matrix factorizes by atom, eq. (5.4), breaks down because collective nature of the system directly implies correlations between the atoms. A new mathematical framework is needed to understand how the system behaves under collective emission and this is an area of future research.

Chapter 6

Conclusion

We explored collective quantum jumps of Rydberg atoms by first performing a probe spectrum and atom-atom cross correlations. The dependence on separation distance as well as geometry was studied. An intermediate case for the collapse operator was introduced allowing us to interpolate between fully independent and fully collective spontaneous emission. Quantum trajectories were simulated utilizing QuTiP Monte Carlo methods (`mcsolve`) for a wide range of separation distances to see how the changing collapse operators inhibited collective jumps. Average jump lengths were calculated to quantify the transition and histograms of excitation population were made to better visualize the optical bistability. We have shown through our analysis that the introduction of collective spontaneous emission suppresses the collective quantum jumps in groups of Rydberg atoms.

Quantum jumps have been an area of interest since the birth of quantum mechanics and still offers interesting research today. Along with the recent developments with respect to the trapping and manipulation of collections of Rydberg atoms, there are many interesting new systems to be explored. My study of the collective quantum jumps of Rydberg atoms is important in understanding the systems that could one day be used in the quantum computing technologies of the future.

6.1 Future Work

There is still work to be done on my model with respect to geometric factors. Along with generalizing to more dimensions other distance dependent variable can be parameterized such as the Rydberg interaction term V. By making the interaction vary based on distance, $V \to V_{ij}$, as one would expect for a dipole-dipole interaction term, interesting behavior is expected. As the atoms move closer together their emission becomes more collective which inhibits the collective jumps but the larger the shift in the energies the more likely there is for

jumps to occur. We want to see for what conditions are jumps still shown as distance is varied and V_{ij} competes with the increasing collective emission effect. The relationship between the inhibition of the jumps as the emission becomes more collective and the enhancement of the jumps as the Rydberg interaction gets stronger is a topic that still needs to be explored. I would like to better characterize the inhibition of jumps to build a better understanding of this transition.

There are still unanswered question for future research such as exactly why the emission type inhibits jumps. Optical bistabilites have been studied for many years and there is further reading that could offer insight. The region of enhanced jump length for $0.3 < R < 1.0$ is an unexplored phenomenon which seems to indicate there is an interesting interaction between the emission type and the strength of the optical bistability in this system.

Appendix A

jumps.py

A main program was made to separate repeated function in able to make varying parameters easier.

```python
import header as hd
import qutip as qt
import numpy as np
import random as rd
import matplotlib.pylab as plt
import itertools
import datetime
import os
import sys
import pickle
from scipy.spatial.distance import pdist
from scipy.spatial.distance import squareform

starttime = datetime.datetime.now()


author   = "Eitan Lees"
save_as   = "test"
pkl_str   = save_as + ".p"
csv_str   = save_as + ".csv"


n = 4
R = 0.001


params = []                        # Parameter list
```

```python
params.append(n)                        # [0] n     - number of atoms
params.append(np.linspace(R, R, 1)) # [1] R     - Seperation Distance
params.append([3.5])                    # [2] dlist - Detuning list
params.append(10)                       # [3] V     - Rydberg shift
params.append(1.5)                      # [4] omega - Driving
params.append('corrected collective')       # [5] Etype - Emission type
params.append(np.linspace(0,100,1000)) # [6] tlist - Time


#raising and lowering ops
sigmaplist, sigmamlist = hd.get_sigma(params[0])


#Building all of the Psi operators
psi = list(itertools.product([0, 1], repeat=params[0]))
psi0, psi_psi = hd.build_psi(params[0], psi)


jumplist = []
op = sigmaplist[0]*sigmamlist[0]
for i in range(1,params[0]):
    op += sigmaplist[i]*sigmamlist[i]
jumplist.append(op/params[0])


# Building Shape and Distance matrix
x_dist = hd.n_line(params[0], params[1][0])
dist_mat = squareform(pdist(x_dist))


# Building Collaps Operators and Shift matrix
collaps = hd.make_collapse(dist_mat, params[5], sigmamlist)
shift = hd.V_edit_rn(dist_mat, 3)


# Building Hamiltonian and running Monte Carlo solver
HAM = hd.hamiltonian(params, sigmaplist, sigmamlist, shift)
all_data = hd.mc_jump(HAM, psi0, params[6], collaps, jumplist)


# Saving data
print ("Let's save the data")
pickle.dump( all_data, open( pkl_str, "wb" ) )
endtime = datetime.datetime.now()
total_time = endtime - starttime
```

```
hd.save_data(params, all_data.expect, csv_str, author, total_time)
print ('DONE')
#
#
# plt.plot(params[6], all_data[0][0])
# plt.show()
```

Appendix B

header.py

A header file was imported into the main program to increase readability. Some functions here are not explicitly used in the main program up were used in earlier versions to do probe spectrum's and other preliminary results.

```python
from __future__ import division
import qutip as qt
import numpy as np
import random as rd
import pylab
import time
import datetime
import matplotlib.pyplot as plt
import itertools


def get_shape(shape, a):
    """
# Returns an array of points for a given shape
# Input : shape - string of shape
#         a     - charateristic distance
#
# Output: x     - np.array of the coordinates
    """
    if shape == "triangle":
        x = np.array([[0, 0], [a, 0],
                      [a / 2, a / 2 * np.sqrt(3)]])
    if shape == "square":
        x = np.array([[0, 0], [0, a],
```

```python
                            [a, a], [a, 0]])
    if shape == "line":
        x = np.array([[0, 1 * a], [0, 2 * a],
                      [0, 3 * a], [0, 4 * a]])
    if shape == "3x3":
        x = np.array([[0, 2 * a], [1 * a, 2 * a], [2 * a, 2 * a],
                      [0, 1 * a], [1 * a, 1 * a], [2 * a, 1 * a],
                      [0, 0 * a], [1 * a, 0 * a], [2 * a, 0 * a]])
    return(x)


def n_line(n, a):
    """
# Returns an array of points for a line of 'n' atoms
# with equadistand seperations of 'a'
# Input : n - string of shape
#         a - charateristic distance
#
# Output: x - np.array of the coordinates
    """
    x = np.zeros((n,2))
    for i in range(n):
        x[i] = [0, i*a]
    return(x)


def get_sigma(n):
    """
# raising and lowering ops
# Input : n          - Number of atoms
#
# Output : sigmaplist - List of sigma plus operators
#          sigmamlist - List of sigma minus operators
    """
    sigmamlist = []
    sigmaplist = []

    for j in range(n):
        if j == 0:
            sigmajm = qt.sigmam()
```

```python
        else:
            sigmajm = qt.qeye(2)
        for i in range(1, n):
            if j == i:
                sigmajm = qt.tensor(qt.sigmam(), sigmajm)
            else:
                sigmajm = qt.tensor(qt.qeye(2), sigmajm)
        sigmamlist.append(sigmajm)
        sigmajp = sigmajm.dag()
        sigmaplist.append(sigmajp)
    return(sigmaplist, sigmamlist)


def G2_ops(P, M):
    top = []
    bottom = []
    for i in range(len(P)):
        for j in range(len(P)):
            if i != j:
                top.append(P[i]*P[j]*M[i]*M[j])
#           else:
#               top.append(P[i]*M[i]*P[j]*M[j])
        bottom.append(P[i]*M[i])
    return(top, bottom)


def build_psi(n, psi):
    """
# Building all of the |Psi><Psi| operators
# Input : n       - Number of atoms
#         psi     - An array of all binary combinations
#
#
# Output : psi0   - Initial State (all down)
#          psi_psi - List of |psi><psi|
    """
    psi_psi = []
    for i in range(0, len(psi)):
        if psi[i][0] == 1:
            x = qt.basis(2, 0)
```

```python
        if psi[i][0] == 0:
            x = qt.basis(2, 1)
        for j in range(1, n):
            if psi[i][j] == 1:
                x = qt.tensor(x, qt.basis(2, 0))
            if psi[i][j] == 0:
                x = qt.tensor(x, qt.basis(2, 1))
        psi_psi.append(x * x.dag())
    psi0 = qt.basis(2, 1)
    for i in range(1, n):
        psi0 = qt.tensor(psi0, qt.basis(2, 1))
    return(psi0, psi_psi)


def V_edit(dist_mat, emission_type, sigmamlist):
    """
# V-Edit
# Input : dist_mat    - Matrix of distance from
#                       atom i to atom j
#         emission_type - "independent"
#                         "collective"
#                         "corrected collective"
#
# Output: shift - Matrix of Rydberg energy shifts
#         collaps - List of collaps operators
    """
    size = len(dist_mat)
    gamma_2 = 1
    d_hat = 1
    lamda = 1
    r_hat = 0
    shift = np.zeros((size, size))
    termA_del = np.zeros((size, size))
    termA_gam = np.zeros((size, size))
    termB_del = np.zeros((size, size))
    termB_gam = np.zeros((size, size))
    xi = 2 * (np.pi) * dist_mat
    for i in range(len(xi)):
        for j in range(len(xi)):
```

```python
            if i != j:
                x = (1 - 3 * (d_hat * r_hat) ** 2)
                termB_del[i][j] = (x * (np.sin(xi[i][j]) / xi[i][j] ** 2
                                 + np.cos(xi[i][j]) / xi[i][j] ** 3))
                termB_gam[i][j] = (x * (np.cos(xi[i][j]) / xi[i][j] ** 2
                                 - np.sin(xi[i][j]) / xi[i][j] ** 3))
                termA_del[i][j] = (-(1 - (d_hat * r_hat) ** 2)
                                 * np.cos(xi[i][j]) / xi[i][j])
                termA_gam[i][j] = ((1 - (d_hat * r_hat) ** 2)
                                 * np.sin(xi[i][j]) / xi[i][j])
    for i in range(size):
        for j in range(size):
            if i != j:
                shift[i, j] = (1/dist_mat[i, j]**6)
    #shift = gamma_2 * (3 / 4) * (termA_del + termB_del)
    Gam = gamma_2 * (3 / 2) * (termA_gam + termB_gam)
    #### Building Collaps Ops with gamma ####
    #### but first we have to fix gamma ####
    np.fill_diagonal(Gam, 1)
    lam, vec = np.linalg.eigh(Gam)
    J = []
    for i in range(len(vec)):
        tot = 0
        for j in range(len(vec)):
            tot += vec[j][i] * sigmamlist[::-1][j]
        J.append((lam[i] ** 0.5) * tot)
    if emission_type == "independent":
        collaps = sigmamlist # Indeptdent
    if emission_type == "collective":
        collaps = sum(sigmamlist) # Collective
    if emission_type == "corrected collective":
        collaps = J # The Gamma correction
    return(shift, collaps)


def make_collapse(dist_mat, emission_type, sigmamlist):
    """
# V-Edit
# Input : dist_mat    - Matrix of distance from
```

```python
#                        atom i to atom j
#          emission_type - "independent"
#                           "collective"
#                           "corrected collective"
#
# Output: shift - Matrix of Rydberg energy shifts
#         collaps - List of collaps operators
    """
    size = len(dist_mat)
    gamma_2 = 1
    d_hat = 1
    lamda = 1
    r_hat = 0
    shift = []
    termA_gam = np.zeros((size, size))
    termB_gam = np.zeros((size, size))
    xi = 2 * (np.pi) * dist_mat
    for i in range(len(xi)):
        for j in range(len(xi)):
            if i != j:
                x = (1 - 3 * (d_hat * r_hat) ** 2)
                termB_gam[i][j] = (x * (np.cos(xi[i][j]) / xi[i][j] ** 2
                                   - np.sin(xi[i][j]) / xi[i][j] ** 3))
                termA_gam[i][j] = ((1 - (d_hat * r_hat) ** 2)
                                   * np.sin(xi[i][j]) / xi[i][j])
    Gam = gamma_2 * (3 / 2) * (termA_gam + termB_gam)
    #### Building Collaps Ops with gamma ####
    #### but first we have to fix gamma ####
    np.fill_diagonal(Gam, 1)
    lam, vec = np.linalg.eigh(Gam)
    J = []
    for i in range(len(vec)):
        tot = 0
        for j in range(len(vec)):
            tot += vec[j][i] * sigmamlist[::-1][j]
        if lam[i] > 1e-12:
            J.append((lam[i] ** 0.5) * tot)
        # else:
```

37

```python
    #     J.append(0 * tot)
    if emission_type == 'independent':
        collaps = sigmamlist # Indeptdent
    if emission_type == 'collective':
        collaps = sum(sigmamlist) # Collective
    if emission_type == 'corrected collective':
        collaps = J # The Gamma correction
    return(collaps)


def V_edit_rn(x,z):
    """

#Returns the shift of 1/dist^n
    """

    size = len(x)
    y = np.zeros((size, size))
    for i in range(size):
        for j in range(size):
            if i != j:
                y[i, j] = (1/x[i, j]**z)
    return(y)


def hamiltonian(params, sigmaplist, sigmamlist, shift):
    n     = params[0] # n    - Number of atoms
    dlist = params[2] # dlist - Detuning list
    V     = params[3] # V    - energy shiftdue to dipole - dipole
    omega = params[4] # Omega - rabi freq


    H = [[] for __ in range(len(dlist))]
    for k, delta in enumerate(dlist):
        hlist = []
        hilist = []
        for i in range(n):
            hx = -delta * (sigmaplist[i] * sigmamlist[i])
            hy = (omega / 2) * (sigmaplist[i] + sigmamlist[i])
            hlist.append(hx + hy)
        for i in range(n):
            for j in range(i):
```

```python
                ####  Note this is where you can turn on and off the shift
                hix = (V) / (n - 1)
                #hix = (V * shift[i][j]) / (n - 1)
                hiy = sigmaplist[j] * sigmamlist[j]
                hiz = sigmaplist[i] * sigmamlist[i]
                hilist.append(hix * hiy * hiz)
        H[k] = sum(hlist) + sum(hilist)
    return(H)


def me_loop(H, psi0, collaps, oplist):
    tlist = np.linspace(0, 10, 100)
    datalist = []
    for i in range(len(oplist)):
        datalist.append([])
    for k in range(len(H)):
        data1 = qt.mesolve(H[k], psi0, tlist, collaps, oplist)
        for i in range(len(oplist)):
            datalist[i].append(data1.expect[i][-1])
    return(datalist)


def mc_loop(H, psi0, collaps, oplist, tlist):
    """
# Monte Carlo master equation solver
# Input : H       - Hamiltonian for our system
#         psi0    - Initial State (all down)
#         collaps - List of collaps operators
#         oplist  - List of expectation values
#         tlist   - time of trajectory
# Output : datalist - List of state evolved through time
    """
    ntraj = [1]
    datalist = []
    for i in range(len(oplist)):
        datalist.append([])
    for k in range(len(H)):
        opts = qt.Options()
        opts.seeds = [12345]
        data1 = qt.mcsolve(H[k], psi0, tlist, collaps, oplist, ntraj, options =
```

```python
            opts)
        for i in range(len(oplist)):
            datalist[i].append(data1.expect[i][-1])
    return(datalist)


def mc_jump(H, psi0, tlist, collaps, oplist):
    """
# Monte Carlo master equation solver
# Input : H       - Hamiltonian for our system
#         psi0    - Initial State (all down)
#         collaps - List of collaps operators
#         oplist  - List of expectation values
#         tlist   - time of trajectory
# Output : datalist - List of state evolved through time
    """
    ntraj = [1]
    opts = qt.Options()
    opts.seeds = [12345]
    results = qt.mcsolve(H[0], psi0, tlist, collaps, oplist, ntraj, options =
        opts)
    return(results)


def sort_raw_data(n, psi, datalist):
    """
# Data Sorting
# Input : n       - Number of atoms
#         psi     - An array of all binary combinations
#         datalist - List of Mesolve data
#
# Output: result - List of |psi|^2 for n excited atoms
    """
    tot_datalist = []
    for i in range(n + 1):
        tot_datalist.append([])
    for i in range(len(psi)):
        x = sum(psi[i])
        for j in range(n + 1):
            if x == j:
```

```python
                tot_datalist[j].append(datalist[i])
    result = []
    for i in range(n + 1):
        result.append([])
    for i in range(len(tot_datalist)):
        result[i] = [sum(x) for x in zip(*tot_datalist[i])]
    return(result)


def quantum_plot(x, y, plot_type, save_str, n, title_r):
    """
#  DEPRECIATED
#
#
# Input : x       - x cordinate, usually time
#         y       - y cordinate, usually |psi|^2
#         plot_type - "full" includes all psi
#                   "atom" only n atoms plotted
#         save_str - string of where to save file
#
# Output: saves file to save_str and shows plot
    """
    ### Now for plotting ###
    fig = plt.figure()
    ax = plt.subplot(111)

    colormap = pylab.cm.gist_rainbow
    ## plotting
    # Specify the colormap to use for plotting
    #################################
    if plot_type == "full":
        ## Full Plot
        pylab.gca().set_color_cycle([
            colormap(i) for i in np.linspace(0, 0.9, len(psi))])

        for i in range(len(psi)):
            ax.plot(dlist, datalist[i], label='$\psi_{%i}$' % i)

        ## Plot total prop
```

```python
        pylab.plot(dlist, total, label='total')


    if plot_type == "atom":
        ### Atom Plot
        pylab.gca().set_color_cycle([
            colormap(i) for i in np.linspace(0, 0.9, len(y))])


        ax = plt.subplot(111)


        for i in range(n + 1):
            ax.plot(x, y[i], label='%i Atoms Excited' % i)


        ### shrinking plot and legend placement
        box = ax.get_position()
        ax.set_position([box.x0, box.y0, box.width * 0.8, box.height])
        ax.legend(ncol=1, loc='right',
                    bbox_to_anchor=[1.4, 0.5],
                    columnspacing=1.0, labelspacing=0.0,
                    handletextpad=0.0, handlelength=1.5,
                    fancybox=True, shadow=True)
        ### I want to add the spacing in the title
        pylab.title("Probability vs. $\delta/\gamma$\nr =
            {:0.3f}".format(title_r))
        pylab.xlabel('$\Delta/\gamma$')
        pylab.ylabel('Probability')
    plt.savefig(save_str)
    plt.close()


def save_data(params, all_data, out_str, author, total_time):
    with open(out_str,'w') as out_file:
        time = datetime.datetime.now()
        line = "# " + author + ", " + time.strftime("%Y-%m-%d %H:%M:%S")+ "\n"
        line += '# Run time : {}'.format(total_time)+ "\n"
        line += '# Parameter list'+ "\n"
        line += '# [0] n    - number of atoms    --> ' + str(params[0])+"\n"
        line += '# [1] R    - Seperation Distance --> ' + str(params[1][-1])+"\n"
        line += '# [2] dlist - Detuning list     --> ' + str(params[2][-1])+"\n"
        line += '# [3] V    - Rydberg shift       --> ' + str(params[3])+"\n"
```

```python
        line += '# [4] omega - Driving        --> ' + str(params[4])+"\n"
        line += '# [5] Etype - Emission type  --> ' + str(params[5])+"\n"
        line += '# [6] tlist - Time           --> ' + "From " + \
        str(params[6][0]) +" to "+ str(params[6][-1])+ "\n"
        out_file.write(line)

        for i in range(len(all_data[0][0])):
            line = ""
            line += str(params[6][i])
            line += "," + str(all_data[0][0][i])
            line += "\n"
            out_file.write(line)
    return


def csv_save_data(params, results, csv_str, author, total_time):
    """ Currently Broken! """
    line = author + ", " + time.strftime("%Y-%m-%d %H:%M:%S")+ "\n"
    line += 'Total Run time : {}'.format(total_time)+ "\n"
    line += 'Parameter list'+ "\n"
    line += '[0] n   - number of atoms   --> ' + str(params[0])+"\n"
    line += '[1] R   - Seperation Distance --> ' + str(params[1][-1])+"\n"
    line += '[2] dlist - Detuning list   --> ' + str(params[2][-1])+"\n"
    line += '[3] V   - Rydberg shift      --> ' + str(params[3])+"\n"
    line += '[4] omega - Driving          --> ' + str(params[4])+"\n"
    line += '[5] Etype - Emission type   --> ' + str(params[5])+"\n"
    line += '[6] tlist - Time             --> ' + "From " + \
    str(params[6][0]) +" to "+ str(params[6][-1])+ "\n"

    np.savetxt(csv_str, results, header = line, delimiter = ",")
```

Appendix C

r-jumps.r

The R programing language was used to do post processing of the quantum jump results. Jump statistics as well as plotting was done within R.

```r
jumpstat <- function(X, Y) {
    count <- 0
    for ( i in 1:(length(X)-1)) {
        # Jumps up
        if (Y[i] < 0.2 && Y[i+1] >= 0.2) {
            starttime <- X[i]
            count <- count + 1
            if (count == 1) { number <- 1 }
            else { number <- c(number, count) }
        }
        # Jumps Down
        if (Y[i] >= 0.2 && Y[i+1] < 0.2){
            if(count == 1){ jumptime <- X[i] - starttime}
            else{jumptime <- c(jumptime, X[i] - starttime)}
        }
    }
    if (count == 0 ){ jumptime <- 0 }
    return(jumptime)
}


pullData <- function(Fname) {
    dat <- read.csv(Fname, skip=10, header=F)
    names(dat) <- c('time', 'energy')
    return(dat)
```

```r
}

histplot <- function(Fname) {
    run <- pullData(Fname)
    runjumps <- jumpstat(X = run$time, Y = run$energy)
    hist(runjumps, freq = F, # breaks=c(0:100),
        main=c(Fname,length(runjumps)),
        xlab="jump Length")
}

jumpplot <-function(Fname) {
    run <- pullData(Fname)
    plot(run, type='l',
        cex.axis=1.5,
        cex.lab=1.5,
        main = strsplit(Fname,".csv|data/")[[1]][2],
        ylab = "<E>",
        ylim=c(0,0.5))
    abline(h=0.2, col = 'red')
}

fig5b <- function(X) {
}

fig5c <- function(X) {
    # This needs work to me made into loops and be more general
    dat <- list(length(X))
    for(i in 1:length(X)) {
        dat[[i]] <- pullData(X[i])$energy
    }
    # Hplt <- list(length(X))
    h1 <- hist(dat[[1]], plot=F)
    h2 <- hist(dat[[2]], plot=F)
    h3 <- hist(dat[[3]], plot=F)
    h3x <- h3$mids
    h3y <- h3$density
    h2x <- h2$mids
    h2y <- h2$density
```

```
    h1x <- h1$mids
    h1y <- h1$density
    plot( h1y ~ h1x,
        type="o",
        col="red",
        pch=0,
        ylim=c(0,40),
        xlab="Excitation Population",
        ylab="Density")
    points( h2y ~ h2x, type="o", col="blue",pch=1)
    points( h3y ~ h3x, type="o", col="green",pch=2)
    legend("topright",
        title="Seperation Distance",
        legend=c(0.1, 0.2,1.0),
        col=c("red","blue","green"),
        lty=c(1,1,1),
        lwd=c(2.5,2.5))
}


bigplot <- function(X) {
    num <- length(X)
    par(mfrow=c(num,1))
    for( i in 1:length(X)) {
        # histplot(X[i])
        jumpplot(X[i])
    }
    par(mfrow=c(1,1))
}


avgjumps <- function(Flist) {
    avgjumps <- vector()
    for( i in 1:length(Flist)) {
        run <- pullData(Flist[i])
        value <- mean(jumpstat(X = run$time, Y = run$energy))
        avgjumps <- c(avgjumps, value)
    }
    return(avgjumps)
}
```

```r
Fnamedat <- function(Flist) {
    Fdat <- vector()
    for(i in 1:length(Flist)) {
    Fdat <- c(Fdat,as.numeric(strsplit(Flist[i],".csv|=")[[1]][2]))
    }
    return(Fdat)
}


avgplot <- function(Flist) {
    x <- Fnamedat(Flist)
    y <- avgjumps(Flist)
    plot(x,y,
        # log="x",
        type="o",
        cex.axis=1.5,
        cex.lab=1.5,
        main='Average jumps length vs R',
        xlab = 'Distance',
        ylab = 'Average Jump Length')
    ind     <- 4.894111
    abline(h=ind, col='red')
}


runFiles <- c(#'data/ind-r=1.0.csv',
        # 'data/r=0.2.csv',
        # 'data/r=0.3.csv',
        # 'data/r=0.4.csv',
        # 'data/r=0.5.csv',
        # 'data/r=0.6.csv',
        # 'data/r=0.7.csv',
        'data/r=0.8.csv')
        # 'data/r=0.9.csv',
        # 'data/r=1.0.csv',
        # 'data/r=10.0.csv',
        # 'data/r=100.0.csv')


# avgplot(runFiles)
```

```
bigplot(runFiles)
# fig5c(runFiles)
```

Appendix D

Mathematica Code

The following code was used to solve and graph the optical Bloch equations for the steady state of our system, eqs. (5.14) and (5.13).

Independent:

```
Manipulate[
 Plot[Evaluate[\[Rho]ee /.
    Quiet@Solve[
      2/\[CapitalOmega]^2 (\[CapitalDelta] -
          V \[Rho]ee)^2 \[Rho]ee + \[Gamma]^2/(2 \[CapitalOmega]^2) \
\[Rho]ee + (\[Rho]ee - 1/2) == 0, \[Rho]ee]], {\[CapitalDelta], 0,
  10}], {{V, 10, "Rydberg Energy Shift"}, 0.1,
 20}, { { \[CapitalOmega], 1.5, "Driving"}, 1,
 5}, { {\[Gamma] , 1, "Gamma"}, 1, 5}]
```

Collective:

```
Manipulate[
 Plot[Evaluate[\[Rho]ee /.
    Quiet@Solve[
      2/\[CapitalOmega]^2 ((\[CapitalDelta] -
           V \[Rho]ee)^2 + \[Gamma]^2/
           4 (2 (myN - 1) \[Rho]ee - myN)^2) \[Rho]ee + (\[Rho]ee -
        1/2) == 0, \[Rho]ee]], {\[CapitalDelta], 0, 10}], {{V, 10,
  "Rydberg Energy Shift"}, 0.1,
 20}, { { \[CapitalOmega], 1.5, "Driving"}, 1,
 5}, { {\[Gamma] , 1, "Gamma"}, 1, 5}, {{myN, 4, "Number of Atoms"},
  Range[1, 16]}]
```

References

[1] E. Urban et al. "Observation of Rydberg blockade between two atoms". In: *Nature Physics* 5 (2009), pp. 110–114.

[2] N. Bohr Dr. phil. "I. On the constitution of atoms and molecules". In: *Philosophical Magazine Series 6* 26.151 (1913), pp. 1–25.

[3] Albert Einstein. "On the quantum theory of radiation". In: *Laser Theory* (1972), pp. 5–21.

[4] M. Bertolotti. *The History of the Laser*. Taylor & Francis, 2004.

[5] W. Neuhauser et al. "Visual observation and optical cooling of electrodynamically contained ions". In: *Applied physics* 17.2 (1978), pp. 123–129.

[6] Warren Nagourney, Jon Sandberg, and Hans Dehmelt. "Shelved optical electron amplifier: Observation of quantum jumps". In: *Phys. Rev. Lett.* 56 (26 June 1986), pp. 2797–2799.

[7] H. J. Carmichael. *Statistical methods in quantum optics*. New York: Springer, 1999.

[8] M. D. Lukin et al. "Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles". In: *Phys. Rev. Lett.* 87 (3 June 2001), p. 037901.

[9] M. Saffman, T. G. Walker, and K. Mølmer. "Quantum information with Rydberg atoms". In: *Rev. Mod. Phys.* 82 (3 Aug. 2010), pp. 2313–2363.

[10] Tony E. Lee, H Häffner, and M. C. Cross. "Collective Quantum Jumps of Rydberg Atoms". In: *Phys. Rev. Lett.* 108 (2012), p. 023602.

[11] A. Urvoy et al. "Strongly Correlated Growth of Rydberg Aggregates in a Vapor Cell". In: *Phys. Rev. Lett.* 114 (20 May 2015), p. 203002.

[12] David Petrosyan, Michael Höning, and Michael Fleischhauer. "Spatial correlations of Rydberg excitations in optically driven atomic ensembles". In: *Phys. Rev. A* 87 (5 May 2013), p. 053414.

[13] Daniel Comparat and Pierre Pillet. "Dipole blockade in a cold Rydberg atomic sample". In: *J. Opt. Soc. Am. B* 27.6 (June 2010), A208–A232.

[14] R. H. Lehmberg. "Radiation from an N-Atom System. I. General Formalism". In: *Phys. Rev. A* 2.3 (Sept. 1970), pp. 883–888.

[15] G. S. Agarwal. "Master-Equation Approach to Spontaneous Emission". In: *Phys. Rev. A* 2 (1970), pp. 2038–2046.

[16] J. R. Johansson, P. D. Nation, and F. Nori. "QuTiP 2: A Python framework for the dynamics of open quantum systems". In: *Comp. Phys. Comm.* 184 (2013), p. 1234.