Masters Defense

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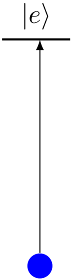
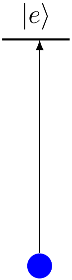
Overview

1. Introduction
2. System
   - Hamiltonian
   - Geometry
   - Quantum Dissipation
3. QuTiP
4. Results
   - Probe Spectrum
   - Cross Correlations
   - Optical Bloch Equations
   - Quantum Jumps
   - Jump Statistics
5. Conclusion

Rydberg Atoms



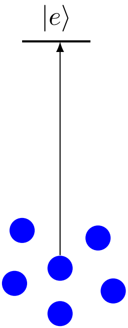
What's a Rydberg Atom?



What's a Rydberg Atom?

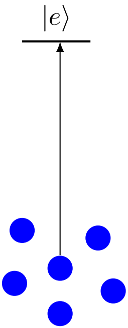
- Highly excited ($n \approx 100$)
- Large dipole moment
- Long range dipole interaction

Multiple Rydberg Atoms



Rydberg interaction for many atoms

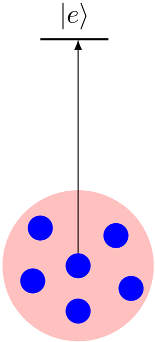
Multiple Rydberg Atoms



Rydberg interaction for many atoms

- Multi atom interaction

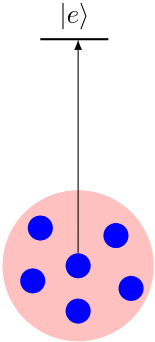
Multiple Rydberg Atoms



Rydberg interaction for many atoms

- Multi atom interaction
- Surrounding atoms "see" dipole

Multiple Rydberg Atoms



Rydberg interaction for many atoms

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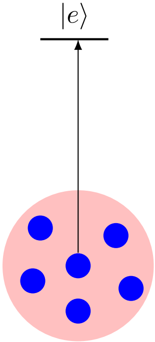


Rydberg interaction for many atoms

- Multi atom interaction
- Surrounding atoms “see” dipole
- Induced energy shift
- Atoms are inhibited from excitation
  $\Rightarrow$ Rydberg Blockade

Current Rydberg Research

Quantum information with Rydberg atoms

M. Saffman and T. G. Walker

Department of Physics, University of Wisconsin, 1150 University Avenue, Madison, Wisconsin 53706, USA

- Quantum Information (Saffman-2010) [1]

Current Rydberg Research

Observation of Rydberg blockade between two atoms

E. Urban, T. A. Johnson, T. Henage, L. Isenhower, D. D. Yavuz, T. G. Walker and M. Saffman*

- Quantum Information (Saffman-2010) [1]
- Observation of Rydberg Blockade (Urban-2009) [2]

Current Rydberg Research

Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles

M. D. Lukin,[1] M. Fleischhauer,[1,2] and R. Cote[3]
[1]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*
[2]*Fachbereich Physik, Universität Kaiserslautern, D-67663 Kaiserslautern, Germany*
[3]*Physics Department, University of Connecticut, Storrs, Connecticut 06269*

- Quantum Information (Saffman-2010) [1]
- Observation of Rydberg Blockade (Urban-2009) [2]
- Mesocopic Atomic Ensembles (Lukin-2001) [3]

Current Rydberg Research

Collective Quantum Jumps of Rydberg Atoms

Tony E. Lee,[1] H. Häffner,[2] and M. C. Cross[1]
[1]*Department of Physics, California Institute of Technology, Pasadena, California 91125, USA*
[2]*Department of Physics, University of California, Berkeley, California 94720, USA*
(Received 29 September 2011; published 9 January 2012)

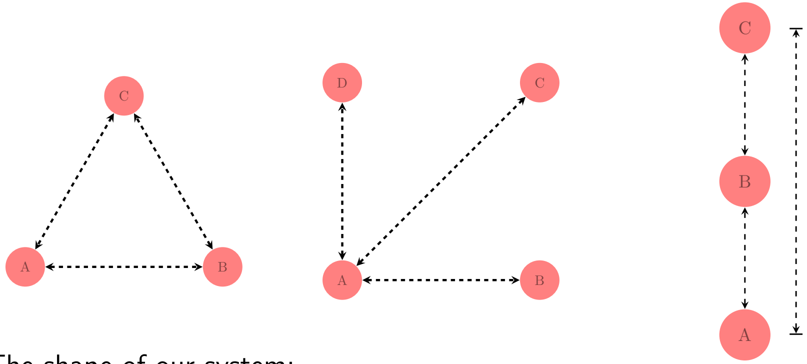
- Quantum Information (Saffman-2010) [1]
- Observation of Rydberg Blockade (Urban-2009) [2]
- Mesocopic Atomic Ensembles (Lukin-2001) [3]
- Collective Quantum Jumps (Lee-2012) [4]

Hamiltonian

$$H = \sum_{j=1}^{N} \left[-\Delta |e\rangle\langle e|_j + \frac{\Omega}{2} (\sigma_{j+} + \sigma_{j-}) \right] + \frac{V}{N-1} \sum_{j<i=1}^{N} |e\rangle\langle e|_j \otimes |e\rangle\langle e|_i \quad (1)$$

- σ_{j-} is the Pauli lowering operator for the jth atom
- Δ is the detuning of the driving laser from the atomic resonance
- Ω is the Rabi frequency
- V is a constant that accounts for the long range interaction between the Rydberg atoms in their excited states.

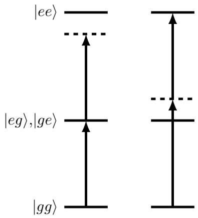
Geometry

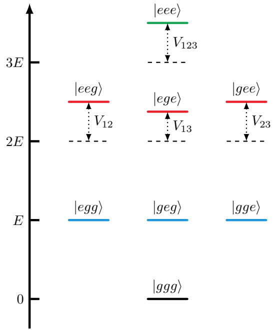


The shape of our system:

- Triangle, square, line ...
- Nearest neighbor interactions
- Lengths varied

Energy Levels





Energy Levels of our system:

- Multi-Photon transitions
- Probe Spectrum and atom-atom cross correlations utilized for insight

Lindblad Superoperator

We introduce dissipation into our system using the Lindblad superoperator

$$\dot{\rho} = \mathcal{L}\rho \tag{2}$$

with

$$\mathcal{L} = \frac{1}{i\hbar}[H_s, \cdot\ ] + \sum_j \frac{\gamma_j}{2}(2\hat{O}_j \cdot \hat{O}_j^\dagger - \hat{O}_j^\dagger \hat{O}_j \cdot - \cdot \hat{O}_j^\dagger \hat{O}_j) \tag{3}$$

which is derived under the Born-Markov approximation:

- Born $\rightarrow$ weak H_{sR} interaction
- Markov $\rightarrow$ reservoir has no memory

Collapse Operators

For our initial simulations we utilize the operator $\hat{O}_j = \sigma_{j-}$, and the master equation,

$$\dot{\rho} = -i\left[H, \rho\right] + \frac{\gamma}{2}\sum_{j=1}^{N}\left(2\sigma_{j-}\rho\sigma_{j+} - \rho\sigma_{j+}\sigma_{j-} - \sigma_{j+}\sigma_{j-}\rho\right) \qquad (4)$$

to describe independent spontaneous emission. We then introduced an altered master equation,

$$\dot{\rho} = -i\left[H, \rho\right] + \frac{\gamma}{2}\left(2J_{-}\rho J_{+} - \rho J_{+}J_{-} - J_{+}J_{-}\rho\right) \qquad (5)$$

to describe collective spontaneous emission where $\hat{O}_j = J_{-} = \sum_{j=1}^{N}\sigma_{j-}$.

Intermediate Collapse Operators

For the intermediate case we use the Lehmberg-Agarwal dipole coupling coefficient

$$\gamma_{ij} = \gamma \frac{3}{2} \left\{ [1 - (\hat{d} \cdot \hat{r}_{ij})^2] \frac{sin\xi_{ij}}{\xi_{ij}} + [1 - 3(\hat{d} \cdot \hat{r}_{ij})^2] \left(\frac{cos\xi_{ij}}{\xi_{ij}^2} - \frac{sin\xi_{ij}}{\xi_{ij}^3} \right) \right\} \quad (6)$$

where

$$\xi_{ij} \equiv k_0 r_{ij} = 2\pi r_{ij}/\lambda_0, \qquad \mathbf{r}_{ij} = r_{ij}\hat{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$$

The eigenvalues, λ, and eigenvectors, $\vec{v}$, where calculated for the γ_{ij} matrix and used to determine the collapse operators for our system by

$$\hat{O}_j = J_i = \sqrt{\lambda_i} \sum_j^N \vec{v}_{ij} \sigma_{j-} \quad (7)$$

Matrix Analysis

$$J_i = \sqrt{\lambda_i} \sum_j^N \vec{v}_{ij} \sigma_{j-}$$

For large r our emission appears independent (Identity Matrix)

$$\begin{pmatrix} 1 & 0 & \dots \\ 0 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \implies \text{all } \lambda_i \to 1$$

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$$\begin{pmatrix} 1 & 0 & \dots \\ 0 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \implies \text{all } \lambda_i \to 1$$

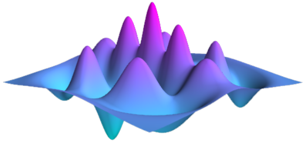
For small r our emission appears collective (Matrix of ones)

$$\begin{pmatrix} 1 & 1 & \dots \\ 1 & 1 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \implies \text{one nonzero } \lambda \to N$$

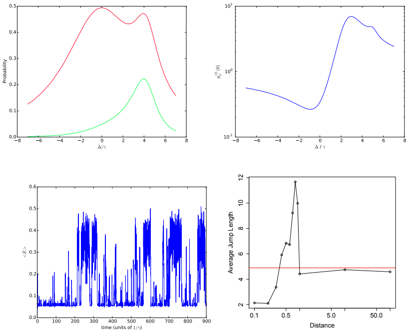
QuTiP

Computational Modeling:

- QuTiP → Quantum Toolbox in Python
- Developed by Paul Nation and Robert Johansson
- In depth quantum framework
  - `mesolve` → Master equation solver
  - `mcsolve` → Monte Carlo Quantum Trajectories
- Open source and under active development at qutip.org



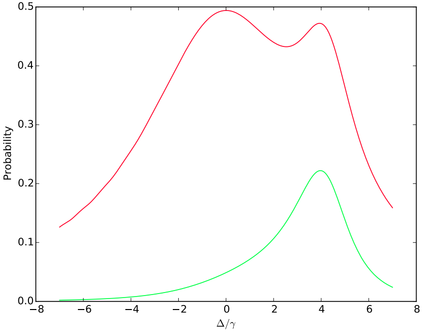
Results



Topics of Research:

- Probe Spectum
- Crosscorelation
- Quantum Jumps
- Jump Statistics

Probe Spectrum: Emission Type



- Probability to excite 1 or 2 atoms for $n = 2$, $\gamma = 1$, $V = 8$, and $\Omega = 4$ for independent emission

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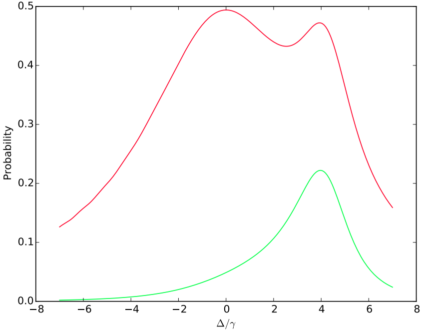
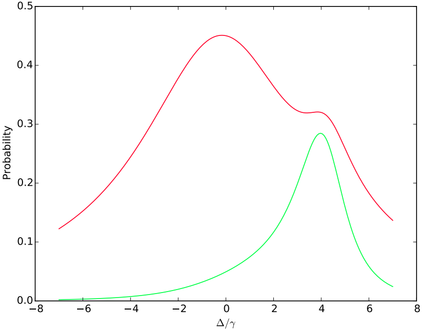
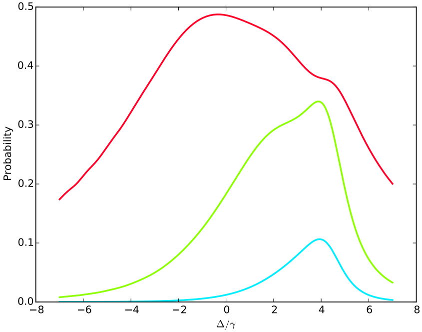




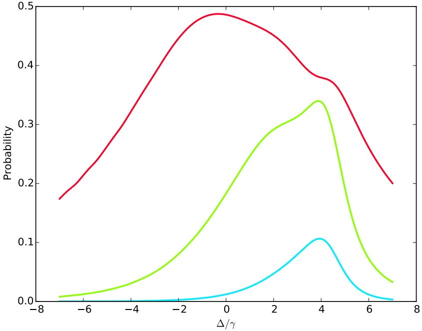
- Probability to excite 1 or 2 atoms for $n = 2$, $\gamma = 1$, $V = 8$, and $\Omega = 4$ for independent emission
- Same parameters, collective emission

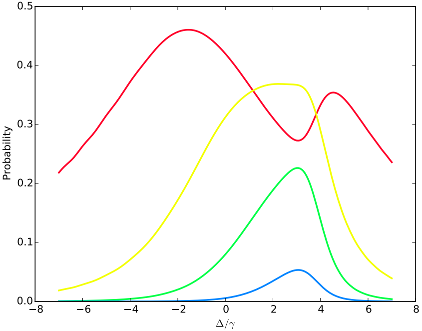
Probe Spectrum: Geometry



- $n = 3$, $\gamma = 1$, $V = 8$, and $\Omega = 4$ for independent emission. The probability for 1, 2, and 3 atoms being excited

Probe Spectrum: Geometry



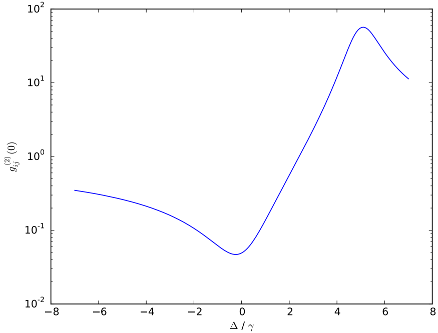


- $n = 3$, $\gamma = 1$, $V = 8$, and $\Omega = 4$ for independent emission. The probability for 1, 2, and 3 atoms being excited
- Same parameters, $n = 4$ Square. The probability for 1, 2, 3, and 4 atoms being excited.

Animations

Animations!

Cross Correlations



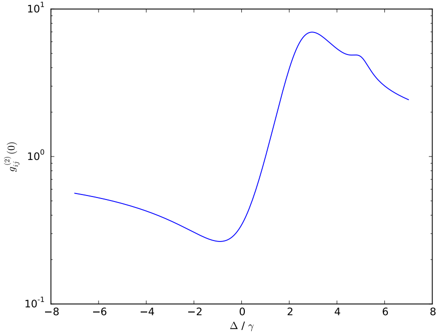
$$g_{ij}^{(2)}(0) = \frac{\langle \sigma_{i+}\sigma_{i-}\sigma_{j+}\sigma_{j-} \rangle}{\langle \sigma_{i+}\sigma_{i-} \rangle \langle \sigma_{j+}\sigma_{j-} \rangle}. \quad (8)$$

We would expect to see a bump at

$$\frac{V}{N-1} \cdot \frac{\#\ \text{Pairs}}{\#\ \text{Photons}}$$

- $n = 2$, $\gamma = 1$, $V = 10$, and $\Omega = 4$ for independent emission

Cross Correlations



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- $n = 3$, $\gamma = 1$, $V = 10$, and $\Omega = 4$ for independent emission
- Same parameters, $n = 3$ Triangle

Mean-Field Theory

A mean-field theory was used to solve the optical Bloch equations

$$\dot{\bar{\rho}}_{ee} = -\Omega \operatorname{Im} \bar{\rho}_{eg} - \gamma \bar{\rho}_{ee}$$
$$\dot{\bar{\rho}}_{eg} = i \left(\Delta - V \bar{\rho}_{ee} \right) \bar{\rho}_{eg} - \frac{\gamma}{2} \bar{\rho}_{eg} + i\Omega \left(\bar{\rho}_{ee} - \frac{1}{2} \right)$$

but it's important to note a major assumption of this approach which is that the density matrix factorizes $\rho = \bigotimes_{j=1}^{N} \rho_j$.

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Independent

$$0 = \frac{2}{\Omega}\left(\Delta - V\bar{\rho}_{ee}\right)^2 \bar{\rho}_{ee} + \frac{\gamma^2}{2\Omega}\bar{\rho}_{ee} + \Omega\left(\bar{\rho}_{ee} - \frac{1}{2}\right) \tag{9}$$

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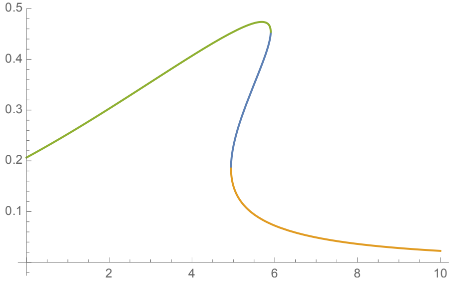
Collective

$$0 = \left(\bar{\rho}_{ee} - \frac{1}{2}\right) + \frac{2}{\Omega^2}\bar{\rho}_{ee}\left[\left(\Delta - V\bar{\rho}_{ee}\right)^2 + \frac{\gamma^2}{4}\left(2(N-1)\bar{\rho}_{ee} - N\right)^2\right] \tag{10}$$

Mathematica Widget

Interactive Mathematica Notebook
(Demo)

Bloch Solutions



- The independent case for $V = 12, \Omega = 3, \gamma = 1$

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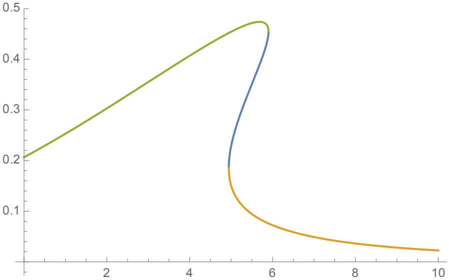
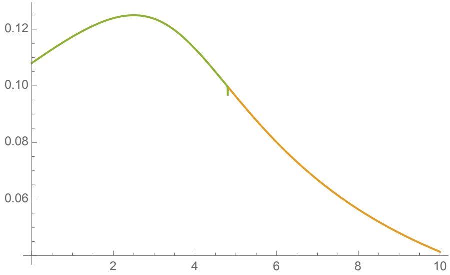
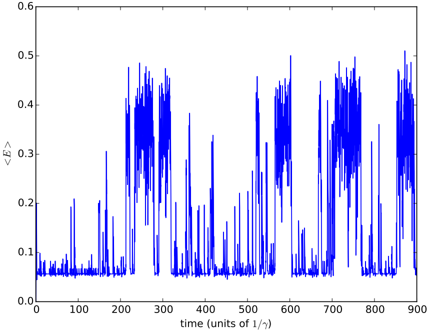




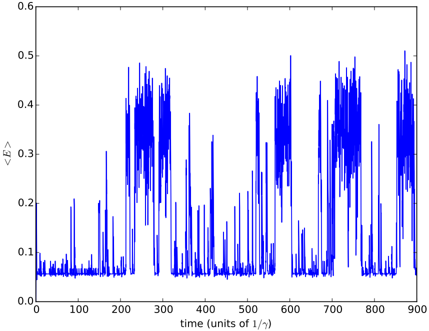
- The independent case for $V = 12, \Omega = 3, \gamma = 1$
- The collective case for $V = 20, \Omega = 5, \gamma = 1, N = 16$

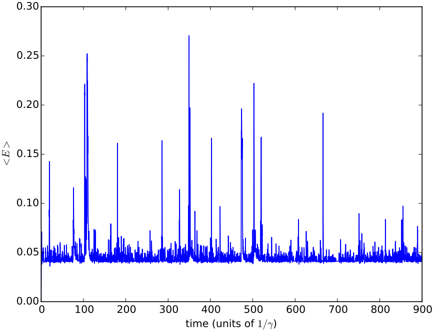
Quantum Jumps



- $n = 12,\ V = 10,\ R = 0.5,\ \Delta/\gamma = 3.5$ for intermediate emission γ_{ij}

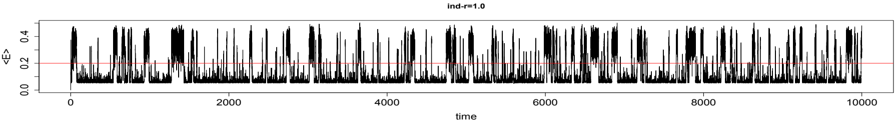
Quantum Jumps





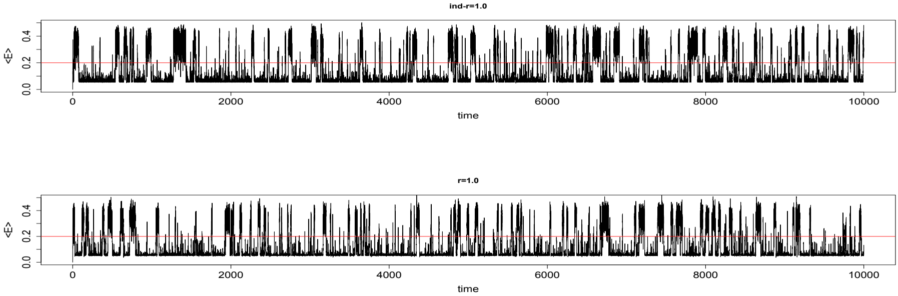
- $n = 12,\ V = 10,\ R = 0.5,\ \Delta/\gamma = 3.5$ for intermediate emission γ_{ij}
- Same parameters, $R = 0.115$

Long Quantum Jumps



- $n = 16$, $V = 10$, $\Omega = 1.5$, $\Delta/\gamma = 3.5$, with intermediate emission γ_{ij}

Long Quantum Jumps



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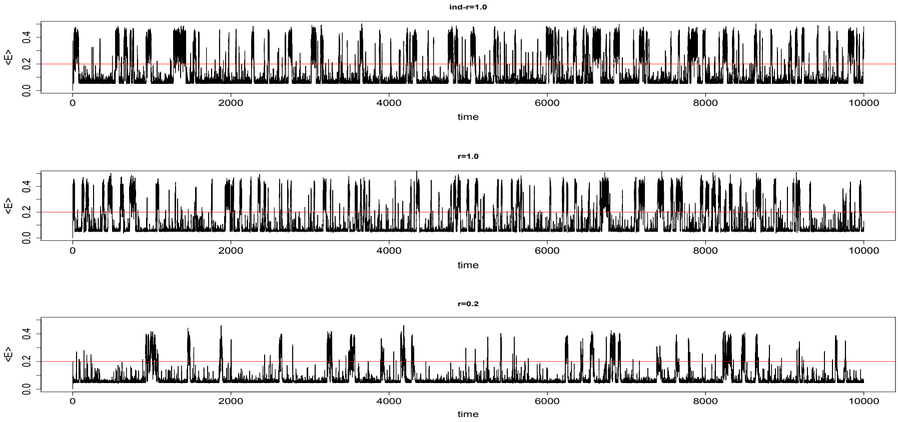

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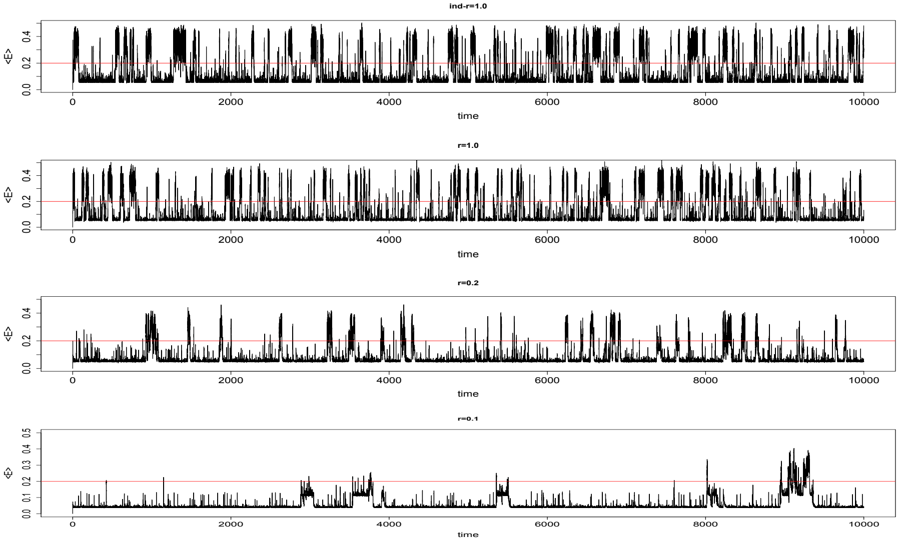
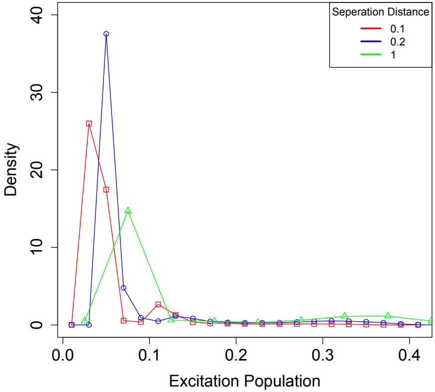


Jump Statistics



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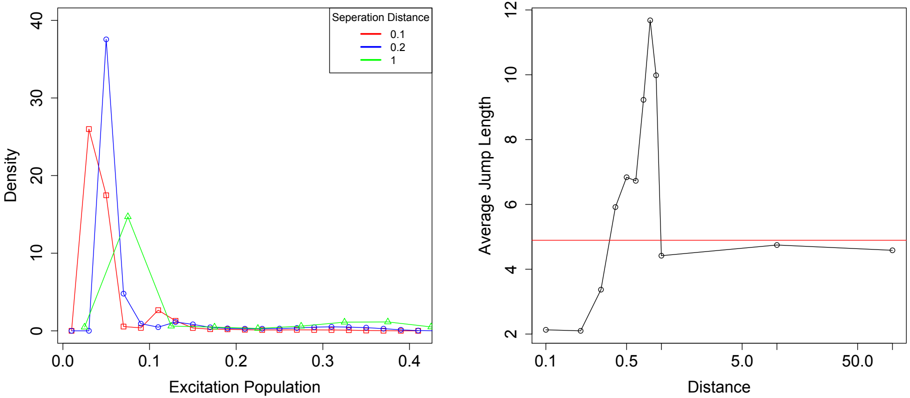

- $n = 16$, $V = 10$, $\Omega = 1.5$, $\Delta/\gamma = 3.5$, with intermediate emission γ_{ij}

Conclusion

Preliminary Analysis:

- Introduced a variable Rydberg interaction V_{ij} and studied probe spectra for different geometric configurations
- Studied atom-atom cross correlations, $g_{ij}^{(2)}(0)$, to look for multiphoton excitations.

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- Reproduced quantum jumps and explored the effects of collective emission
- Explored the effects of a tunable emission type γ_{ij} and saw jumps diminished for short seperations distances

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Future research:

- Explain enhanced jump length for $0.3 < R < 1.0$
- Further characterize jump transition
  - Oscillatory behavior of γ_{ij}
- Explain why collective emission turns off jumps
  - Examine assumptions made in analysis

Thanks!

- James Clemens for much needed advising and research help.
- Perry Rice and Samir Bali for being on my committee and teaching great courses.
- The entire Miami Physics Department and graduate students for help and support.
- Any Questions?

References

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